

Plato, Zeno, Parmenides, and Frege

Plato's *Parmenides* is supposed to be a grand antinomy composed of many little ones. And, in fact, it does anticipate by twenty-four centuries some of the problems and paradoxes of logic and set theory of the turn of our century. That any interpretation of the *Parmenides* is bound to be controversial can be inferred from the diversity of interpretations that have been proposed, from the Neo-Platonic interpretation to the joke interpretation to the now dominant interpretations that find a serious and reasonable purpose for the dialogue as a whole.²

The dialogue proper opens with Socrates' questioning of Zeno's conclusions about plurality. Socrates rebuts Zeno by appealing to the theory of forms. "What is there surprising in someone pointing out that I am one thing and also many?", he asks. "But if anyone can prove that what is simply Unity itself is many or that Plurality itself is one, then I shall begin to be surprised" (129b-c). That's his challenge.

What are these arguments of Zeno's about plurality of which we are given only a hint? Although this is an equally controversial question, it

1 Departamento de Filosofia da PUC-Rio.

2 This paper was delivered, in essentially its present form, at a conference on Antinomies at the Universidade Federal do Rio Grande do Sul, in October 1988.

A basic reference that I will use, and to which I will refer simply as 'Cornford', is F. M. Cornford *Plato and Parmenides* (New York: Bobbs-Merrill, 1939). Two other important papers are G. Ryle "Plato's *Parmenides*" (1939) and W. G. Runciman, "Plato's *Parmenides*" (1959), both reprinted in R. E. Allen (Ed.) *Studies in Plato's Metaphysics* (London: Routledge, 1965). To these should be added G. E. L. Owen "Notes on Ryle's Plato", in Ryle (edited by G. P. Wood and G. Pitcher, New York: Doubleday, 1970). A more recent paper is J. M. Moravcsik "Forms and Dialectic in the Second Half of the *Parmenides*", in M. Schofield and M. C. Nussbaum (eds.), *Language and Logos* (Cambridge: Cambridge University Press, 1982).

must be an important piece in the drama that is about to unfold, and it is a good place to begin.

The interpretation of Zeno that I like is Owen's.³ It is appealing because it integrates the arguments about plurality with the arguments about motion into a coherent whole, and it makes sense of Zeno's assertion to Socrates as to the purpose of his treatise. "This book," he says, "is a retort against those who assert a plurality. It pays them back in the same coin with something to spare, and aims at showing that, on a thorough examination, their own supposition that there is a plurality leads to even more absurd consequences than the hypothesis of the One" (128c-d).

There are two alternatives to begin with; that reality is one and indivisible, as Parmenides asserted, and that reality is divisible, which is the butt of Zeno's attack. If reality is divisible, then either (A) it is infinitely divisible, or (B) it is finitely divisible.

If reality is infinitely divisible, then either (A1) this is a completed divisibility, i.e., as a process it comes to an end, or (A2) it is a potential divisibility, i.e., it never comes to an end.

If (A1), then there are units of some sort, at bottom, that are indivisible, and that either have no size at all or have some size. Then, as Owen shows, Zeno has a whole battery of arguments to ridicule both alternatives. If units have no size, then how can they make a difference either by addition or by subtraction of them to anything? Even infinitely many of them would add up to nothing. So, if reality is divided everywhere in this way, it comes to nothing. If, on the other hand, units have a constant size, then infinitely many of them would add up to infinite size, and anything so divided would be infinitely large. Moreover, if units have size, why can't they be divided any further? Here Owen appeals to the argument of the Stadium to show that they could be divided—and this would apply, he argues, even if the size were infinitely small.⁴ Moreover, the Arrow can also be brought to bear, in either alternative.

It is interesting to note that who saved the day here was not Newton, or Leibniz, with the calculus, but Cantor, with transfinite set theory. It is only because of Cantor's distinction of infinite powers that in modern

3 G. E. L. Owen, "Zeno and the Mathematicians" (1957), reprinted in R. E. Allen and D. J. Furley (Eds.) *Studies in Presocratic Philosophy*, vol. 2 (London: Routledge, 1975).

4 Owen, *Ibid.*, pp. 150-151.

measure theory we can define a measure on the continuum which assigns measure zero to points, and to any finite or denumerably infinite set of them, but that can assign non-zero measures, in a reasonable and consistent way, to sets of the same cardinality as the continuum itself. Thus, in agreement with Zeno's claim, the addition or subtraction of a point, or even of denumerably many points, makes no measurable difference. Yet, if we have *enough* points, then they can add up to make a measurable difference.⁵

If (A2), then the process of division is a potential process that is incompletable. How, then, can Achilles catch up to the tortoise? So, the Achilles and the Dichotomy are Zeno's answers to (A2).

Alternative (B) fares no better at Zeno's hands. To begin with, if we have a finite collection of units, what separates them and makes them distinct? We would need more things, and more, and more, *ad infinitum*. But, anyhow, with this finite divisibility we also have units that either have no size or have some size. If they have no size, then we are just as badly off as in the infinite case —worse, in fact, because not even Cantor can save us. If they do have size, then the Stadium and the Arrow can be used to show that this escape route is also problematic.

It is impressive and it makes sense. But it is not clear, as Socrates points out, that this sort of argumentation can be brought to bear on the division of reality in terms of Plato's theory of forms. That's the challenge that is taken up by Parmenides.

He begins with a direct attack on the theory, raising three fundamental questions. What forms are there? That is, for what kinds of things are there forms? (130b-130d) What is the nature of the forms and of their relation to particulars and to each other? (130e-133b) How is knowledge of the forms possible? (133b-134e)

As to the first question, Socrates has no satisfactory answer, and Parmenides lets him get away with a small admonition. It is a fundamental objection, though, and we see that in modern times natural attempts to close the gap led to more trouble than people could handle —if we assume, for instance, that for any well-defined condition there is a form an-

5 One still has to show that the Arrow (and the other arguments about motion) can be handled in this line. On these questions see A. Grünbaum *Modern Science and Zeno's Paradoxes* (Middletown, Conn.: Wesleyan University Press, 1967), especially Chapter 3.

swering to that condition.⁶ It isn't really a question of Man versus Mud, as Socrates seems to believe, but of scientific and philosophical theories and method. An investigation into the nature of reality is not to be based on common opinion (130e).

The second question, and its development, is supposed to contain Parmenides' main attack on the theory of forms. It also begins to take up Socrates' challenge.

Is participation a sharing? Is the form as a whole in each particular that shares in it? Wouldn't it be divided, then? Could it be like a sail over many things at once? But couldn't we then divide it, on this spatial model? Does it make sense to talk about sharing more or less of a form? What about forms such as Equality, Smallness, Largeness? Would sharing of a small part of Smallness make things larger than those that share of a large part of Smallness? (131a-e)

And, going back to the first question, suppose that it is some sort of comprehension or abstraction principle that leads us to postulate the forms. If the form also shares in this characteristic that we use for comprehension, wouldn't then a second form appear? And a third? *Ad infinitum*, so that each form is again a plurality? (132a-b)

Now comes the kind of move that Frege most disliked. Couldn't forms be thoughts that exist in our minds? "A subjective or an objective thought?", Frege would ask.⁷ "A thought of nothing or a thought of something?", asks Parmenides. If a thought of something, wouldn't then the forms be the objects of thought rather than the thoughts themselves? And if they were thoughts, what about the things that share in them; would they consist of thoughts? Would they think? (132b-c)

So much for sharing. What about forms as patterns and participation as imaging or likeness? The argument here is the Third Man again. It is supposed by many interpreters that the whole attack against the theory of forms hangs on this argument, and that, therefore, if a way out is found, then the attack is seriously blunted. But this argument is just as straightforward as anything that came before it. If Likeness is symmetric, and if the principle of comprehension operates by postulating a separate form

⁶ If the condition can apply to any entities, not just to particulars, and if we don't distinguish types, then we get the Heterodoxical paradox, which is a version of Russell's paradox.

⁷ *The Foundations of Arithmetic* (Oxford: Blackwell, 1950), sections 26 and 27.

for a plurality of things that are alike in some respect, which accounts for their being alike in that respect, then by taking those things plus the postulated form and applying comprehension again, we get a new separate form. And so on, *ad infinitum*. Of course, we can start adding structure to the theory in various ways to see what happens, but no structure is added in the dialogue itself. In the earlier discussion, Socrates looks for ways out; here he lets it slide with a "So it seems".⁸

We come now to the third question, that is, in effect, related to the negative conclusions reached so far but that depends on exploiting the main explanatory feature of the theory of forms; namely, their avowed perfection and exaltedness over the particulars that participate in them. That was the point of Socrates' challenge. So, Parmenides argues, the forms have their being and "are what they are with reference to one another," whereas the particulars "are related among themselves, and not to the forms." (133c-d) He illustrates this point by arguing that a human master is a master of a human slave, "whereas Mastership itself is what it is of Slavery itself." Applying this to knowledge, he concludes that Knowledge itself is related to Reality itself and that our knowledge is related to things in our world. So, how could we know the forms? (133d-134c)

This argument is supposed to be, in Cornford's words, "almost grossly fallacious" (p.98). Yet, when contemporary philosophers come out with clearly related versions of the argument, they are applauded for their subtlety. Benacerraf, for instance, argues that all our knowledge involves causal relations, and that, therefore, since we don't have causal relations to the forms, or to abstract mathematical entities, we won't have knowledge of them.⁹ Neither argument seems valid to me, but they are not grossly fallacious either in that they raise an important question that has to be answered. Given the difference in nature of forms and men, how is knowledge of forms possible for men? For numbers, we can put the question as in Warren McCulloch's title: "What Is a Number, that a Man May Know It, and a Man, that He May Know a Number?"¹⁰ Besides, after all it was Socrates himself who started this kind of talk and emphasized the separation

8 I am not trying to make light of the Third Man argument. What seems to me is that Plato did not intend to examine these arguments here.

9 P. Benacerraf "Mathematical Truth", *The Journal of Philosophy* 70 (1973). Benacerraf refers to Plato's theory of recollection as an attempted way out (p. 675).

10 Reprinted in W. S. McCulloch *Embodiments of Mind* (Cambridge, Mass.: MIT, 1965).

of the world of forms —of which we partake, no doubt, but we saw where that got us— and the world of men. (129a-130a)

We might expect now a little knife twisting in the wound; but no, what Parmenides does is to proclaim that the theory of forms is true. He grants that the objections he has raised have some weight and that “it is extraordinarily difficult to convert the objector.” “Only a man of exceptional gifts will be able to see that a Form, or essence just by itself, does exist in each case; and it will require someone still more remarkable to discover it and to instruct another who has thoroughly examined all these difficulties.”

Moreover, he claims that only the forms give permanence to thought and make significant discourse possible. (135a-135c)

What is the point of this? As I see it, Parmenides is declaring Plato to be the philosophical heir of the Way of Truth. In fact, of course, it is Plato himself who is doing the declaring and acknowledging Parmenides as his master and mentor. He acknowledges this in many ways. The whole *mise en scene* of the dialogue is one of respect and reverence. The ensuing discussion with Socrates places Parmenides at an entirely different level than Socrates in wisdom and assurance. And, after all the objections, it is Parmenides himself who comes out and declares that the theory is sound. It is sound and necessary for thinking and for significant discourse. The strict connection between thought and reality was one of Parmenides’ main theses that Plato is transposing here to his own analysis of reality.

But the fun is only about to begin. Parmenides has hinted several times that what Socrates is lacking is philosophical method. He emphasizes this once more, though now at length. (135c-136e) What was Parmenides’ method? The Goddess tells him: “It is meet that thou shouldst learn all things —both the unmoved heart of rounded Truth and what seems to mortals, in which there is no true belief.”¹¹ He presented his theory of reality in the Way of Truth, and rejected what seems to mortals in the Way of Nothing and the Way of Seeming. Zeno did his bit here, and Plato goes out of his way to point this out both in 128a-e and in 135d-136c. And it is in the second part of the dialogue that the lesson begins.

This is, of course, the difficult part to interpret and it is where interpretations have a hard time. Is Plato just making a fool of Parmenides? The first part just set him up, in the manner of the pompous sophists, and

11 Parmenides, frag. 1, 28-30. (Cornford, p. 30.)

now he is to play the role of buffoon as Plato makes mincemeat of his Way of Truth?¹² Is Plato criticizing him in a nicer way, as Cornford thinks, defining and re-defining "One" to show that all sorts of contradictory things can be asserted of its different senses?¹³ Is Plato talking about God? The Good? Unity? What is going on? Obviously, the fact that there are so many interpretations, many of which with fair evidence in their favor, all of which more or less consistent with the language, gives good reason to think that no interpretation will be entirely smooth and uncontroversial. Nevertheless, I will propose a different view which I find appealing.

The second part of the *Parmenides* consists of eight hypotheses that divide into two main groups; the negative hypotheses (I, IV, VI, VIII), in which all the attributes considered are denied, and the positive hypotheses (II, III, V, VII), in which all the attributes considered are asserted. (There is also a hypothesis IIA, which Cornford views as an appendix to II, that doesn't quite fit into this scheme and whose purpose I shall consider later.) This classification, into negative and positive, refers to the conclusions that are reached and not to the initial formulation of the hypotheses themselves. The first four hypotheses start with an assertion of being, whereas the last four start with an assertion of non-being.

My view is that all the hypotheses are concerned with the theory of forms as presented by Socrates at the beginning, and with his challenge to Zeno; that the negative hypotheses have a certain relation to Parmenides' Way of Nothing, and that the positive hypotheses have a certain relation to the Way of Seeming. But in both cases the subject is the theory of forms and, especially, the form of Unity. The main point of the exercise is to show that if we are too finicky about the interconnections of the forms, then we get nothing; and that if we try to escape this by being more generous, then we get everything, one way or another.¹⁴

12 This is the Burnett-Taylor interpretation. Cornford (p. 114) gives the following quotation from P. H. Frye: "Considered light-heartedly, the *Parmenides* is one of the funniest things in philosophy—the youthful Socrates, the future champion of sound sense and right reason, taking a lesson, open-mouthed—in ontological rigmarole from the old Eleatic dialectician, in the company of Zeno, the subtle juggler of apory and paradox!" *Plato* (Nebraska, 1938), p. 28.

13 Cornford, pp. 105-107, 109-115.

14 The idea that Parmenides is taking up Socrates' challenge and, therefore, that he is discussing the theory of forms, and Unity, is common to many interpretations—for instance, those of Ryle, Runciman and Moravcsik in the papers cited above. What is new, as far as I know, is the connection that I make between the *Parmenides* and Parmenides' three Ways,

This complements the first part of the dialogue by showing, on the one hand, that Socrates' purist conception of Unity as just Unity, Plurality as just Plurality, Likeness as just Likeness, *etc.*, is untenable; because it leads, eventually, to the denial of even that, and of being. And, on the other hand, it shows that with some ingenuity, and a few shifts of meaning here and there, anything goes. ("Henceforward learn what seems to mortals, hearkening to the deceitful order of my words.")¹⁵

Although the first part of the dialogue ends with an acknowledgment of the soundness of the theory of forms, the difficulties that were raised are real difficulties and they show that the theory is in need of more systematic formulation. It was too simplistic and allegorical; more a picture than a theory. The second part emphasizes the difficulties but at the same time opens the way for a better, more reasoned, formulation. Unfortunately, in spite of many efforts, Plato never got a really adequate formulation.

One of the things that struck me when I first studied the *Parmenides* was a certain difference in character of the argumentation in the negative hypotheses and the argumentation in the positive hypotheses. In Hyp. I, for example, it is possible to interpret all the arguments in a fairly uniform way. That is, the arguments are not difficult in general, there are no shifts of meaning in the words used, and they are basically straightforward arguments without really fishy steps. The shortcomings that we may find in some of them can also be found in many other arguments of Plato's. My point, in general, is that the arguments in the negative hypotheses admit of a nice and smooth interpretation.

This picture is changed in the positive hypotheses. In Hyp. II, for example, the task of interpretation is much harder and so are the arguments. (It is also obvious that many of the more interesting arguments are in the positive hypotheses, especially Hyp. II.) To give a uniform interpretation here seems impossible, because in order to have coherent arguments one must allow for shifts in the meaning of the words. Cornford points out a number of places in which this happens. Moreover, these shifts of meaning are not only relative to the use of the terms in Hyp. I, but they also take place within Hyp. II. At the same time we find Plato telling us that

and the specific connections that I will point out between the various hypotheses, *Parmenides'* Ways, and the arguments in the first part of the dialogue.

¹⁵ *Parmenides*, frag. 8, 51-52. (Cornford, p. 45.)

“however many times you utter the same word, you must always mean the same thing.” (147d-e) If this is meant seriously, then either one has to find a way through Hyp. II which avoids the shifts of meaning, or else one has to find an explanation for these shifts that preserves the coherence of the individual arguments, and that gives a purpose to Hyp. II as a whole. One way of sticking to the first alternative is to declare that many of the arguments are really fallacious and that the point is to find those fallacies. There may be some truth in this *vis a vis* some of the arguments, but it seems to me that it wasn't clear to Plato himself how to sort out the wheat from the chaff; and so, as a *tour de force*, he decided to formulate arguments for all opposite conclusions.

Let me try now to give some flesh to the view I am proposing. Before I begin, though, I should point out that in order to make any interpretation of the *Parmenides* stick, even temporarily, one should go through the whole dialogue very carefully analyzing the arguments and their interconnections. I do this briefly for Hyp. I, but to do it for the whole dialogue in detail would require an entire book.

I said before that the theory of forms is Plato's heritage from Parmenides. Not evidently in the actual distinctions of the theory, but in the outlook that its structuring of reality presupposes. There seems to be rather general agreement that the primitive basis of the theory derives from Socrates, and that it was developed into a more systematic ontology in Plato's middle dialogues. It became his Way of Truth and it allowed him to make a synthesis of the philosophies of Socrates, Parmenides and Heraclitus. The theory, however, kept its Socratic basis according to which each form was just what it was and nothing else. They were pure essences, immutable as Parmenides' Being, unchangeable and unperishable, that did not allow for more or less nor for contrary characters. The forms are, and cannot not be, and they are just what they are. It is this picture that the young Socrates puts forward in his challenge to Zeno, and he repeats it several times over to make sure that it sinks in.

The first part of the dialogue does not confront this challenge directly; it works around it. It shows that something has to be done to formulate the theory on a sound basis beyond its Socratic beginnings. It recognizes the worth of the theory, and its necessity, to escape the challenge of the sophists and of Heraclitus. What is to be done? Parmenides certainly points out several fundamental problems in the argument with Socrates. One must get clear as to what forms there are, what is the nature of their relation to particulars and

to each other, and how knowledge of them is possible. No doubt Plato could have said a lot more about these questions in the first part of the dialogue, but he did not want to. He did not see his way clearly through the problems and there was no point in confusing the issues. As it is the problems are there, clearly stated, and Socrates insecurity toward them is evident to anyone. It is also evident that Socrates does not buy the conclusions, and neither does Parmenides —they are weighty difficulties, to be sure, but they are to be surmounted by philosophical analysis.

Several interpreters have conjectured that the first and second parts of the *Parmenides* may have been written at different times. I would speculate that the first part may have been written after the second, as an introduction to it.¹⁶ My reason for this is that if Plato had wanted to deal directly with the difficulties, that were probably in the air for some time, he would and could have done it in detail. Why didn't he bring in the theory of *anamnesis*, for example? Why didn't he pursue the conception of forms as patterns and of participation as imaging or likeness? Even for the conception of participation as sharing, Socrates lets Parmenides get away with the shift from the analogy of the day to the analogy of the sail. ("Don't you think that is a fair analogy?" Parmenides asks. "Perhaps it is," replies Socrates. (131b-c)) Why? There was no point, that's why. Plato undoubtedly had some answers, but he didn't have really satisfactory answers. Interestingly enough, we find the same problem in Frege. Why didn't Frege ever pursue the solution to the paradoxes that he wrote to Russell and published as an appendix to vol. 2 of *The Basic Laws of Arithmetic*? Because he wasn't convinced; the problems ran deep and he didn't really see his way through them.¹⁷

16 In his 1963 "Afterword" to "Plato's *Parmenides*", Ryle argues that the two parts may "have been composed at different, and even quite distant dates," but that "Plato cannot have composed either Part with the intention that it should be the complement, inside one dialogue, of the other," because "while Part I of the *Parmenides* is in *Oratio Obliqua*, Part II, apart from one initial 'he said', is in *Oratio Recta*" (p. 145). He goes on to question even the thematic connection that he had established between the two parts. He repeats these claims, and goes much further, in *Plato's Progress* (Cambridge: Cambridge University Press, 1966). Owen replies to this in "Notes on Ryle's Plato", p. 371.

17 See Frege's correspondence with Russell in G. Frege *Philosophical and Mathematical Correspondence* (Oxford: Blackwell, 1980), and "Appendix II" in G. Frege *The Basic Laws of Arithmetic* (translated and edited by M. Furth, Berkeley: University of California Press, 1964). W. V. Quine discusses some of the problems in "Frege's Way Out", *Mind*, 1955. Frege's solution was revived by Hintikka in the 1950's but the resulting systems were shown to be inconsistent —see A. A. Fraenkel and Y. Bar-Hillel *Foundations of Set Theory* (Amsterdam: North-Holland, 1958).

The *Parmenides* marks Plato's transformation into a full-fledged methodologist. What he had done before was too easy, too poetical, too allegorical. He could stir deep emotions, and bring tears to our eyes. He was a master of criticism, as was Frege after him, and he could expose and debunk those pompous asses that thought they had knowledge. But that was not enough. What was his theory of reality? What was his theory of knowledge? What would Frege be if we only kept the criticism and the allegory? Plato may well be admonishing himself when he says that "there is something noble and inspired in your passion for argument; but you must make an effort and submit yourself, while you are still young, to a severer training in what the world calls idle talk and condemns as useless." (135D) The Plato of the *Theaetetus*, the *Sophist*, and the *Philebus*, for example, is another Plato, digging deeper and not so preoccupied with effect. And so is the Plato in the second part of the *Parmenides*.

What are the forms? What are their relations to one another? What are their relations to particulars? How do we attain knowledge of them? Where are the answers to be found? Shall we appeal to the Gods? Was it the Goddess' revelation that gave knowledge to Parmenides? Not even the Gods can help because their knowledge is not of this world; it is Knowledge itself of Reality itself. (134C-E) One must seek knowledge by examining the opinions of mortals, even though they "wander, two-headed; for perplexity guides the wandering thought in their breasts."¹⁸ This examining is not just collecting; it is discerning, arguing, pursuing the opinions to their ultimate consequences. That is philosophical method.

Hyp. I will begin to take up Socrates' challenge to Zeno by examining the purist conception of forms —the early backbone of the theory. It is initially the first of Socrates' proposals that will be subjected to this examination; the form as a whole and participation as sharing. Then it moves forward a bit. I will go through the whole of Hyp. I, with a few comments here and there, and a few rearrangements and abbreviations.

If Unity is what it is, it cannot be many. [As Socrates had claimed.] Therefore, it cannot have parts, because it would be a plurality of these parts. And it cannot be a whole either, because, by definition, a whole is that from which no part is missing. By having no parts, it will have neither beginning, middle,

18 *Parmenides*, frag. 6, 5-6. (Cornford, p. 32.)

nor end; therefore, it will have no limits. It will not have a shape, because all shapes partake of round and/or straight and, since round is that of which the extremes are equidistant from the center, and straight is that of which the middle intercepts the view of the extremes, anything that partakes of round or of straight or of a mixture of both would have parts and would be many. (137c-138a)

Unity cannot be anywhere. To be in anything is to be surrounded by that thing. If it were in another it would be surrounded by that in which it was, and would touch it in many places and with many parts. If it were in itself, it would be surrounding none other than itself and it would be two; that which surrounds and that which is surrounded. (138a-b)

It cannot change in nature because when it changes, and ceases to be itself, it can no longer be Unity. It cannot move round in the same place because it would have to have parts that revolve around a middle. It cannot change in place either because we have already shown that it cannot be in anything, and still less can it come to be in anything. Anything that comes to be in anything, can neither as yet be in that other thing, while still coming to be, nor be altogether out of it, if already coming to be in it. Therefore, whatever comes to be in another must have parts, and then one part may be in, and another part out of the other. And it cannot come to be as a whole either because it is not a whole. If it were quiet or at rest, on the other hand, it would have to be in the same. But if it were in the same, it would be in that thing, namely the same (in which it is), but we saw that it cannot be in itself or in anything else. Therefore, it cannot change in nature, it cannot change in place, and it cannot be at rest. (138b-139b)

So far the arguments have been quite straightforward, and the conclusions are compatible with the view put forward by Socrates. In particular, the notion of part has always been used in a fairly literal sense that includes the portions into which a body may be divided (in practice or in thought), the units of a number, the parts of a geometrical figure, including those of less dimension. Nowhere was it necessary to interpret 'part' as including characters or properties, as suggested by Cornford.¹⁹ What is being rejected is the quasi-physical, spatial, conception of the nature of the forms suggested by the analogy of the day, or of the sail.

19 Cornford interprets 137C-D as *defining* "the One" as something that has no properties whatever —i.e., 'part' is interpreted as 'characteristic' in the widest sense.

Further, Unity cannot be either the same as itself or as another, or other than itself or than another. If it were other than itself, it would not be Unity; if it were the same as another, it would be that other and not itself. Neither will it be other than another, as long as it is Unity. Because not Unity, but only Otherness, and nothing else, can be other than something. Then not by virtue of being Unity will it be other. But if not by virtue of being Unity, not by virtue of itself; and if not by virtue of itself, not itself, and itself not being other at all, it will not be other than anything. Neither can Unity be the same as itself. The nature of Unity is surely not that of Sameness. For it is not the case that whenever a thing becomes the same as anything it becomes Unity. Anything that becomes the same as Plurality, becomes Plurality and not Unity. But, if there were no difference between Unity and Sameness, when a thing became the same it would always become one; and when it became one, the same. Therefore, if Unity is to be the same as itself, it will not be one as itself, and will therefore be one and also not one. (139b-e)

The point of the last two arguments is that only by sharing of Sameness or Otherness can Unity be the same as itself or other than something. If the forms are distinguished apart just by themselves and don't share in one another, as Socrates maintains about Likeness and Unlikeness, Unity and Plurality, Rest and Motion (129d-e), then he has to accept these conclusions. Unity's strip-tease has begun.

Nor can Unity be like or unlike anything, whether itself or another. What is like itself or another is that which is affected in the same way as itself or another. But we saw that Sameness was of a distinct nature than Unity, and if Unity had any affection apart from being Unity, it would be affected as to be more than one, which is impossible. What is unlike itself or another is that which is affected in a different way than itself or another. But Unity cannot be affected in a different way, for in that case it would be so affected as to be more than one. (139e-140b)

Again, being of this nature, Unity can neither be equal nor unequal either to itself or to another. If it is equal, it is of the same measures as that to which it is equal. And if greater or less than things which are commensurable with it, Unity will have more measures than that which is less, and fewer than that which is greater. And of things that are not commensurable with it, Unity will have greater measures than that which is less and smaller than that which is greater. But what does not partake of Sameness cannot have either the same

measures or anything else the same. And not having the same measures, Unity cannot be equal either to itself or to another. And, again, whether it had fewer or more measures, it would have as many parts as it has measures; and thus, once more, it would be no longer one but as many as its measures. And if it were of one measure, it would be equal to that measure; yet it has been shown to be incapable of equality. Then it will neither partake of one measure, nor of many, nor of few, nor of the same at all, nor be equal to itself or to another; nor be greater or less than itself or another. (140b-d)

These two arguments have a very clear logical structure and are almost corollaries of the previous conclusions. It should be noticed that the notion of part was used again in the same narrow sense of the first few arguments. The next argument is also a straightforward corollary.

Can Unity be older or younger than anything, or of the same age with anything? Whatever is of the same age with itself or another must partake of equality and likeness of time; but Unity does not partake either of Equality or of Likeness. And since it does not partake of Inequality or Unlikeness either, it cannot be older or younger, or of the same age with, anything. Therefore, Unity cannot be older or younger, or of the same age, either with itself or with another. (140e-141a)

So much, in particular, for the idea that the forms, in the purist conception, are thoughts in the mind. Thoughts, after all, are temporal entities of which we can say that they are older or younger than, or of the same age as, other thoughts, or themselves. The next argument will cinch this by showing that Unity cannot be in time at all.

Whatever is in time, must be always becoming older than itself. And that which is older, must always be older than something which is younger. Then, that which becomes older than itself, also becomes at the same time younger than itself, if it is to have something to become older than. I mean this. A thing does not need to become different from a thing which is already different; it is different. And from a thing that has become different, it has become different. And from a thing that will be different, it will be different. But from that which is becoming different, it cannot have been, or be about to be, or yet be, different; the only possibility is that it is *becoming* different. But elderness is a difference with something younger, and nothing else. Then that

which becomes older than itself must also, at the same time, become younger than itself. And it cannot become for a longer or shorter time than itself; it must become, and be, and have become, and be about to be, for the same time with itself. Then things which are in time, and partake of time, must be of the same age with themselves, and must also become at once older and younger than themselves. But Unity did not partake of these affections. Then it does not partake of time, and is not in time. (141a-d)

This is, to my mind, the most interesting argument of Hyp. I. If we take a thought, for instance, or an intuitionistic construction, then once it is introduced in time, it begins to age; it becomes older. But older than what? Older than itself, at the moment of introduction. But this is not a process that is completed now, or has been completed in the past, or will be completed in the future; it is a process of becoming, not of being. The *difference* is becoming; and so, at the same time as the thought or construction becomes older than itself, it also becomes younger than itself. But, since it is at the same time —it is one process, not two —it is of the same age as itself. It is curious that mathematical intuitionists and other mental constructivists who like to emphasize the temporal character of their constructions, do not worry very much about what happens to their constructions as beings in time.²⁰

Well, but do not the expressions “was”, and “has become”, and “was becoming”, signify participation in past time? And do not “will be”, “will become”, “will have become”, signify a participation in future time? And “is”, and “is becoming”, signify a participation in present time? And if Unity has no participation whatsoever in time, it never has become, nor was becoming, or

20 Brouwer says, for instance (L. E. J. Brouwer “Points and Spaces”, *Canadian Journal of Mathematics*, 1954, p. 2), that the first act of intuitionism “recognizes that mathematics is a languageless activity of the mind having its origin in the basic phenomenon of the perception of a *move of time*, which is the falling apart of a life moment into two distinct things, one of which gives way to the other, but is retained by memory. If the two-ity thus born is divested of all quality, there remains the common substratum of all two-ities, the mental creation of the *empty two-ity*. This empty two-ity and the two unities of which it is composed, constitute the *basic mathematical systems*. And the basic operation of mathematical construction is the *mental creation of the two-ity of two mathematical systems previously acquired*, and the consideration of this two-ity as a new mathematical system”. This is closely related to our present discussion, and to the construction of numbers in 142d-144a of Hyp. II, but Brouwer does not examine the temporal features of his construction.

was, at any time, or becomes now, or is becoming, or is, or will become, or will have become, or will be, hereafter. But there are no modes of partaking of Being other than these. Then Unity cannot possibly partake of Being, and Unity is not at all. Then Unity is not, even to the extent as to be Unity; for if it were, and partook of Being, it would already be; but if the argument is to be trusted, Unity neither is one, nor is. Then there is no name, nor expression, nor perception, nor opinion, nor knowledge of it. Then it is neither named, nor expressed, nor opined, nor known, nor does anything that is perceive it. (141d-142a)

So, starting from Socrates' conception, we seem to be landed in the Way of Nothing. Yet, the final argument, where the *coup de grace* is given, is not quite convincing—and this may be the point of the qualification “if the argument is to be trusted.” It is shown that Socrates' initial conceptions, a quasi-physical conception and a mental conception, do not seem to hold much water. But Unity could have Being, albeit atemporal being. The forms are supposed to be eternal and immutable, and perhaps their being is of a different sort, in some sense. Why can't we hang on to the view that Unity is just that *and* has Being?

This is the point of Hyp. II (142b-155e). It starts by showing that one can then claim reasonably that Unity is a plurality, and goes on to make mincemeat of Socrates' challenge to Zeno. Moreover, once we start on this road there seems to be no good place to stop, because we can produce many good arguments—though some better than others—to show that Unity has all the characteristics and affections that were denied in Hyp. I. We are landed in the Way of Seeming, where perplexity reigns. This seems to destroy the purist conception of forms, and is basically a Zenonian exercise. But many of the arguments in this hypothesis are also specifically related to the various suggestions that are put forward by Socrates in the initial conversation with Parmenides. Part of the point is to show what can sustain these views of the nature of forms.

There is, however, another alternative; an attempt to eat our cake and have it too. It seems to be related to the view of forms as patterns that are fixed in the nature of things. One could argue that as a pattern Unity is such as Socrates described it; a pure essence. But, as it manifests itself in things it partakes of their mutable nature and enters into all sorts of relations with the other forms; including, of course, contrary forms. Imagine, for instance, that we have two pieces of clay and that we join them into

one. What happened here? Well, we had Unity in each of them, and Duality, somehow, in both. Now Duality has departed to its pure state, and Unity manifests itself into their union. And when we separate the clay again into two pieces, then Unity separates into many and Duality comes back. This is where Hyp. IIA comes in.

"Let's take up the argument yet a third time," begins Parmenides, and he proceeds to describe Unity as having the characteristics of the virginal Unity of Hyp. I and of the promiscuous Unity of Hyp. II. He supposes then that it shifts from one state to the other coming to be and ceasing to be; it is generated and destroyed. And since it is both one and many, when it becomes one its existence as many is destroyed, and when it becomes many its existence as one is destroyed. And in becoming one and many it is separated and combined. And when it becomes like and unlike, it is assimilated and dissimilated. And so we come to motion and rest and to the conclusion that the transition from one to the other must be instantaneous. (155e-156d)

And what is this strange thing, the instant? One must suppose that whatever it was that Zeno read at the beginning of the meeting, the instant was a big villain. In fact, the argument that follows is very much in the spirit of Zeno's Arrow.²¹

It does not change from rest while it is still at rest, nor from motion while it is still moving; but there is this strange instantaneous nature, something interposed between motion and rest, not existing in any time, and into this and out from this that which is in motion changes into rest and that which is at rest changes into motion. Then Unity, if it is at rest and in motion, must change in each direction. But in changing, it changes instantaneously, and when it changes it can be in no time, and at that instant it will be neither in motion nor at rest. And when it changes from being to destruction or from not being to becoming, does it not pass into an intermediate stage so that it neither is nor is not, neither comes to be nor is destroyed? And when it passes from one to many or from many to one, it is neither one nor many, it is neither separate nor combined? [And so on.] (156d-159b)

So, we are back in the Way of Nothing. Cornford's interpretation of

²¹ Ryle also points out that "this looks like a variant of a Zenonian paradox about motion" ("Plato's *Parmenides*", p. 126).

Hyp. IIA seems to me quite absurd. He is so concerned in rejecting the Neo-Platonic and the Hegelian interpretations, that he does not see that IIA has no point as a corollary to II. Besides, it is quite clear that IIA begins by considering the results of the first two hypotheses.²²

Hyp. III (157b-159b) follows up on Hyp. II and develops the same kind of conclusions for the others; *i.e.*, particulars and forms other than Unity. Similarly, Hyp. IV (159b-160b) develops the conclusions of Hyp. I for the others.

Why four more hypotheses? Isn't this enough?²³ Well, what has been done so far is to explore Being, both from the point of Socrates' original formulation (I, IV), and from the more generous point of view (II, III) — and the mixed attempt IIA. Hyp. I and IV landed us in the Way of Nothing; and so did IIA. Instead of trying to get out of the conclusions of Hyp. I by arguing that Unity is just what it is and has Being, one could try to argue that Unity is just what it is; period. It doesn't have Being. This is the Way of Nothing, and, according to Parmenides, mortals do get tempted by it. So much so, that they talk of what is not and profess knowledge of it. And Hyp. V and VII will show us how they do it. For the Socratic doctrine, however, this is no way out, as will be shown in Hyp. VI and VIII.

The point of Hyp. VI is to show that if Unity does not share in Being, then it has no characters whatsoever; it cannot be anything at all. There cannot be any knowledge, or opinion, or perception of it. It cannot have a name ("Unity"), nor be the subject of discourse. (164b) Whatever may have been fishy about the last argument in Hyp. I, the argument is made much tighter with the explicit hypothesis that Unity does not share in Being. And similarly for the others in Hyp. VIII.

22 Cornford says: "We are starting from the result which has just been reached at the end of that deduction [Hyp. II]: a one thing which exists and becomes in time." If it were a different hypothesis, it "would destroy the symmetry of the whole set of Hypotheses" (p. 194). Yet, he translates 155E as follows (p. 196): "To take up the argument yet a third time: if there is a One such as we have described — a One which is both one and many and neither one nor many and is in time — it follows that since it is one, it has existence at some time; and again since it is *not* one, at some time it has not existence." It doesn't seem very plausible, even with his italics. The same goes for other interpretations that I have mentioned, except for Owen's. Everybody seems to agree that a separate IIA would spoil the logic. Runciman, for instance, who has many interesting insights, doesn't even comment on this question. In "Notes on Ryle's Plato" Owen views IIA as an appendix to I and II, but he analyses it differently than I do.

23 This is, again, a question for which the answers are somewhat contrived.

This is, then, my picture of the *Parmenides*. It is based mainly on internal evidence and on some general considerations about Plato's earlier theory of forms, Parmenides, and Zeno. It seems to me to make sense of the structure of the dialogue in all respects. The choice of characters, the setting, Socrates' youth, the initial conversation with Zeno, Parmenides' cross-examination of Socrates, his praise for the theory of forms, his advice on method, and the dialectical exercise itself. Moreover, it makes sense of the development of Plato's later philosophy, and, especially, of the fact that although he never abandoned the theory of forms, he never put it forward in the same categorical and simple-minded way. It makes clear sense, for instance, of the discussion of forms and of dialectical method in *Sophist* 248a-259d and in *Philebus* 13a-18d.²⁴

What about Plato's various criticisms of Parmenides in the *Sophist*? Doesn't that conflict with my view and give force to the idea that in the *Parmenides* itself Plato is criticizing Parmenides? I don't think so. Plato's identification with Parmenides as to the fundamental aspects of his outlook on reality, does not make Plato a *follower* of Parmenides. In fact, by sharing a basic insight, it makes them also share many problems that this insight brings with it. It is quite true that the problems of the theory of forms reflect back on Parmenides' conception of reality, and that many of the arguments in the *Parmenides* apply to Parmenides' own view; and

24 This is common to various interpretations, and it is one of the main virtues in interpreting the *Parmenides* as a serious consideration of the theory of forms. A big problem, however, is the interpretation of the *Timaeus*, which is generally considered to be later than the *Parmenides*, and where the theory of forms is introduced according to the patterns and likeness conception (*Timaeus* 27d-29d, 31a-b, 33b-34b). Owen tried to get around this by means of a fairly radical and ingenious restructuring of Plato's work in "The Place of the *Timaeus* in Plato's Dialogues" (1953, reprinted in Allen, *Op. Cit.*), according to which the *Timaeus* is earlier than the *Parmenides*, and drew a long reply by H. F. Cherniss, "The Relation of the *Timaeus* to Plato's Later Dialogues" (1957, also reprinted in Allen). I am not competent to evaluate this controversy, but according to the comments I have read it seems that Cherniss got the upper hand — see, e.g., Runciman p. 152. But, in fact, even the *Timaeus* being later than the *Parmenides* does not seem to me to spoil these interpretations concerning the development of Plato's theory of forms. The conception of forms as patterns, and of participation as likeness, does not really come under heavy attack in the first part of the *Parmenides*. And the attack that I have presumed to find in Hyp. IIA is by no means definitive. If we remember, moreover, that the Gods are supposed to have knowledge of the forms (137c), and that the builder of the *Timaeus* is a God, Plato could certainly have built his cosmology in this way simply by leaving aside the argument that the Gods don't have knowledge of our world — which is sacrilegious anyway. This is, of course, sheer speculation, but it seems to me to make some sense.

Plato points this out directly and straightforwardly in *Sophist* 244b-245e. He goes beyond, however, in *Sophist* 258c-259d.

But Plato never managed to solve the problems that he raised to his own satisfaction. Who took the first decisive step toward a solution was Frege. Frege's hierarchy of functions and objects is the first coherent attempt to structure the logical and mathematical forms.²⁵ It is interesting to see what happens with Unity, Plurality, and the other cardinality forms in Frege's hierarchy.

In *The Foundations of Arithmetic*, Frege argues convincingly that cardinality properties are not properties of objects. Why not? Precisely for the reasons that Zeno is supposed to have pointed out; objects would be one and many. This happens, concludes Frege, because there are no units in objects, and not because, as Socrates supposes (139c-d), both Unity and Plurality are instantiated in objects. Unity and Plurality are instantiated in forms, i.e., in Frege's terminology, in concepts of objects. The concept Man, for instance, brings a unit with it; or, rather, it is a unit —Man applies to individual men and not to parts of men.²⁶ Yet, although Man is a unit, Unity is not instantiated in Man, i.e., it does not qualify (or apply to) Man; what qualifies Man is Plurality. And Plurality applies to Man precisely because Man, in virtue of its unit, applies to many things. But isn't Socrates *one* man? Yes, but this means nothing in addition to Socrates being *a* man; i.e., to Man being instantiated in, or applying to, Socrates. And that *two* men are arguing about forms means that Duality applies to the concept Men-arguing-about-forms.

Moreover, although Unity is the same as itself, and is different from Plurality, Unity does not apply to itself. What applies to (level 2) Unity is (level 3) Plurality, because there are *many* (level 1) concepts that apply to *one* thing. And, by the same token, (level 3) Unity does not apply to (level 2) Unity. And so on, for all the Unities and Pluralities, *ad infinitum*. There must be infinitely many of them, of course, by an argument very much like the Third Man argument and a generalization of Frege's earlier considerations. But isn't (level 2) Unity *one* level 2 concept? Yes it is, and what

25 I am not suggesting, however, as Ryle initially did, that Plato had some glimmerings of these modern logical developments—but it was a valuable contribution of Ryle's paper to connect the two.

26 *The Foundations of Arithmetic*, section 46.

this means is that (level 3) Self-identity for level 2 concepts applies to this Unity.

Being (or Existence) does not apply to objects either, and neither does it apply to itself. (Level 2) Existence applies to Man, however, and (level 3) Existence applies to (level 2) Unity; but what applies to (level 2) Existence is (level 3) Plurality. Of course, also (level 3) Existence applies to (level 2) Existence, but this is not what is normally meant when one says that (level 2) Existence is. The latter is captured by (level 3) Self-Identity applying to (level 2) Existence. But Reality as such neither is nor is not; and it is neither one nor many. In fact, none of the concepts in the hierarchy apply to Reality —i.e., to the hierarchy as a whole.

Frege did not discuss directly the problems that Plato raises in the first part of the *Parmenides*, but he did discuss them in various places in a way that is reminiscent of Plato's.²⁷ And he also discussed them in a more modern way inherited from Galileo and from Newton, but which derives from Aristotle's axiomatic method. He tried to discern and formulate the laws of the structure of forms. Since he was concerned with the logico-mathematical forms, it is for those that his laws are particularly relevant. These laws are embodied in his logic, which has a direct interpretation in the hierarchy of levels.

But he did give some fairly straightforward answers to Parmenides' questions in his more philosophical works. Parmenides' first question, as to which characteristics of things determine forms, was answered by Frege in terms of the comprehension (or abstraction) principle for concepts; any well-defined condition determines a concept.²⁸ This principle is quite

27 As far as I know, there isn't a single reference to Plato in Frege's published works. Frege was initially inspired by Leibniz, and rationalism; but rationalism has its roots. In Frege's *Posthumous Writings* (Chicago: University of Chicago Press, 1979), there are two references to Plato. The first (p. 203) is a fairly general reference where Frege claims that "if one counts logic as part of philosophy, there will be a specially close bond between mathematics and philosophy, and this is confirmed by the history of these sciences (Plato, Descartes, Leibniz, Newton, Kant)." The second (p. 253) is a quite specific and very interesting remark on one of Frege's important insights. He says that in seeking a better foundation for mathematics "I soon realized that number is not a heap, a series of things, nor a property of a heap either, but that in stating a number which we have arrived at as the result of counting we are making a statement about a concept (Plato, *The Greater Hippias*)." Presumably Frege is referring to *Greater Hippias* 301d-302b.

28 If one respects differences in level, this abstraction principle is not problematic; as Frege points out in his letter to Russell of 22 June 1902 (*Correspondence*, pp. 132-133).

clearly present, and plays an important role, in his formulations in *The Foundations of Arithmetic*.

As to the nature of forms, Frege agrees with Plato that they are eternal, immutable, atemporal, separate entities. They are of a nature entirely different from the nature of objects, including here physical objects, mental objects and even the ideal objects of mathematics —though the latter are also eternal, immutable, atemporal, etc. This difference is partly reflected in Frege's conception of the unsaturatedness of concepts.²⁹ Admittedly, this conception of saturatedness and unsaturatedness is somewhat allegorical; but it is an answer, and it serves to explain also the combination of concept and object as a process of saturation that is Reality.³⁰ This is his conception of truth that leads him to Parmenides' and Plato's preoccupation with thought, judgment, and knowledge.

Furthermore, Frege agrees with Plato that knowledge of the forms is obtained by reason and thought.³¹ His logical system of deduction, and the deductions therein, are Frege's relative answer to the question about knowledge: If you grant me knowledge of some basic logical forms, then I'll show you how to obtain knowledge of all the logico-mathematical forms. And he did.

Where Frege went wrong was in his attempt to assimilate the mathematical *objects* to *extensions* of concepts. He shared Plato's dislike of units³² —which, more likely than not, was an effect of Zeno's criticisms— and tried to do better. It didn't work, for reasons that are already glimmering in the Third Man argument. One can hardly blame him for trying. And, as he says: *Solatium miseris, socios habuisse malorum*.³³

29 All this is quite clear in *The Foundations of Arithmetic*. Frege faces some of the consequences of his view that "the concept Horse" refers to an object and not to a concept, because of saturatedness, in "On Concept and Object" (in *Translations from the Philosophical Writings of Gottlob Frege*, edited by P. Geach and M. Black, Oxford: Blackwell, 1960). Some of the moves in this paper are reminiscent in style to Plato's. In particular, there is a certain amount of complaining about the limitations of language.

30 See Frege "On Sense and Reference", in Geach and Black, *op. cit.*

31 *The Foundations of Arithmetic*, section 36, for instance.

32 Frege *Ibid.*, sections 29-33; Plato *Republic* 525d.

33 *The Basic Laws of Arithmetic*, App. II. p. 127. Frege is referring to Cantor, Russell, Dedekind, etc.