Kant on Indirect Proofs

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In the second half of the first *Critique*, called Transcendental Method, Kant made various important remarks concerning the discipline of pure reason in working with indirect proofs. One rule of this discipline says that pure reason is not allowed to employ apagogical proofs, but must always proceed by ostensive proofs (B 817). An ostensive or direct proof, in every kind of knowledge, is « that which combines with the conviction of the truth of the conclusion insight into the sources of its truth » (*ibid.*), that is, into the truth of its premises. An apagogical or indirect proof, on the other hand, is not based on such an insight, but on purely formal rules, namely, on the rule of *modus tollens* (which says that if a single false consequence can be drawn from a proposition, the proposition itself is false, B 819) and on the principle of the excluded middle. When we prove a judgment apagogically we require « only to show that a single one of the consequences resulting from its opposite is false, in order to prove that this opposite itself is false, and that the proposition which we have had to prove is therefore true » (*ibid.*). This technique is also known as *reductio ad absurdum*.

Kant had precise grounds to reject the principle of the excluded middle with predicate negation as it is generally formulated. In arguments about the magnitude of the world, which constitute the first two antinomies, this principle leads to contradictions. It is therefore to be expected that Kant would also reject the universal applicability of the apagogical method. In comments on the rule of discipline for proofs quoted above, Kant actually takes this step and advances a general proof-theoretical argument in favour of this disciplinary measure.

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The main reason adduced for rejecting the method of *reductio ad absurdum* as generally applicable is that it does not enable us to gain insight into sources of the truth of the conclusion, or, as he puts it, into « its connection with the ground of its possibility » (B 817). For, Kant argues, in proving a judgment by *reductio ad absurdum* we do not review, as in an ostensive proof, the whole sequence of grounds that can lead us to the truth of this judgment. This method of proof is therefore to be regarded rather as a last resort and notas a mode of procedure which satisfies all requirements of reason » (B 818). The first and the most fundamental of all requirements of reason in the matter of proving synthetic judgments is that we must establish the objective validity of the concepts employed as well as the possibility of their synthesis (B 810). This requirement applies to synthetic propositions in general. In proofs of all of them, we must step « beyond the concept of the object »; and we can do so « only with help of some special guidance, supplied from outside the concept » (*ibid.*). In agreement with Kant's general theory of our cognitive powers, there are just two sources of knowledge which can be helpful here : the pure and the empirical intuition. We can now say that apagogical proofs do not satisfy the Kantian fundamental requirement, because they are based upon purely formal rules of the understanding and do not take into account any intuitive guidance in addition to concepts.

In the light of the fundamental requirement, no attempt to prove by indirect method a synthetic a priori judgment in which ideas of pure speculative reason occur can be tolerated. As Kant explains, the « transcendental enterprises of pure reason are one and all carried on within the domain proper to dialectical illusion, that is, within the domain of the subjective, which in its premisses presents itself to reason, nay, forces itself upon reason, as being objective » (B 820). We know already which are the premisses that force themselves upon us as being objective or constitutive and thus induce us into errors of subreption. They are either analytic principles of traditional formal logic or synthetic transcendental principles, such as the principles of transcendental realism. As we have shown elsewhere[2], transcendental realism is characterized by the dialectical illusion which believes that by taking the mentioned synthetic principles as premises, we can derive, from entirely unobjectionable data and by applying traditional formal logic, *new* synthetic knowledge about the sensible world. The moral of the antinomies is that we cannot. No pure logic (formal principles plus *a priori* semantics), the Kantian no more than the realist, is capable to reveal any aspect whatsoever of the structure of the sensible world.

If the indirect method does not satisfy all requirements of our reason, is it an admissible method of proof at all ? Kant's answer to this question is that the apagogical method can be applied « only in those sciences where it is

2   See Loparic' 1990.

impossible to substitute mistakenly what is subjective in our representations
for what is objective, that is, for the knowledge of that which is in the object »
(B 819). Traditional metaphysics is not such a science. We have shown above
that the Kantian antinomies arise from certain premises accepted as true by
transcendental realism. These realist premises are deceptive because they
operate with concepts either *objectively impossible* or *entirely inconceivable*.

The former case is illustrated by the first two antinomies. In the first
antinomy, for instance, the deceptive premise is the one which says that the
sensible world is *given* as complete in its magnitude or as determined in its
totality. This assertion is erroneous because it employs an objectively impos-
sible concept, namely, the concept of sensible world as a complete *quantum* in
itself. This concept is objectively impossible because its referent, the sensible
world itself, cannot possibly be complete. For it is given only through the
empirical regress *in indefinitum,* a procedure which cannot possibly generate
complete totalities nor exemplify any « determinate concept ». The realist
sensible world is therefore an impossible entity, a non entity. To such a case,
says Kant, applies the rule : *non entis nulla sunt predicata;* that is, all that is
asserted of a non-entity, whether affirmatively or negatively, is erroneous.
Consequently we cannot arrive at knowledge of the truth of a judgment
through refutation of its logical opposite (B 820-1). In other words, the indirect
method does not apply. This methodological point is the key to the Kantian
solution of the first two antinomies.

False semantical principles of transcendental realism lead to various other
objectively impossible concepts. The very concept of reality of transcendental
realism is self-contradictory and must also be expurgated.[3] We see on this
example that long before the commonly agreed beginning of analytical
philosophy metaphysics itself was dealt with in a logical way.

Examples of logically possible, but entirely inconceivable concepts are
provided by the two last antinomies. In them occur the unconceivable concepts
of liberty and of necessary being. The realist is right in not accepting as
objectively true judgments which contain them. He commits an error, however,
when he denies the possibility of inconceivable things themselves. For in-
stance, the unconditioned necessity in the existence of an entity is subjectively
inconceivable to us (B 820). We become guilty of a subreption, however, if for
that reason we deny that such a supreme entity is possible in itself. Here again
the error of the realist consists in forcing upon things themselves our subjective
conditions of conceiving them. Our subjective incapacity to conceive of
something is no better measure of things than our subjective capacity to think
of it.

Notice that the recognition of the self-contradictory character of concepts
of transcendental realism cannot in general be gained directly by analysis. Kant

3   Cf. *Logik Bromberg,* AA, vol. 24.2, p. 261; *Logik Busolt, ibid.,* p. 655).

himself arrives at it by studying antinomies, that is, from consequences which can be derived from it. The contradictions are not obtained by directly applying the principle of non-contradiction but indirectly, by showing that some propositions do not obey the principle of sufficient reason[4].

The general result of these considerations is that subjective conditions of intuition and of thought can creep into our objective thought in cases where we do deal with objects as given to us, which happens in all metaphysical questions. For that reason, we are forbidden to employ the apagogical method of proof in this science.[5] Since the apagogical method does not satisfy the fundamental requirement on proofs of synthetic judgments, the question arises if it is a permissible method at all. Are there sciences in which subreptions of the kinds just specified are impossible ?

There are such sciences and one of them is mathematics. On Kant's view, traditional mathematics construes or at least can always construe all its concepts in pure intuition by avoiding all logical and, in general, all discursive construction procedures and employing only intuitive ones. That is why in this science « all our conclusions can be drawn immediately from pure intuitions » (B 810-11) and why it can be said that, as it stands, it satisfies the Kantian requirement on proofs of synthetic propositions.

If this is so, why do mathematicians so frequently resort to indirect proofs ? According to Kant, there are two reasons for that practice. Firstly, indirect proofs are generally more convincing than direct proofs. Contradiction, says Kant, « always carries with it more clearness of representation than the best connection, and so approximates the intuitional certainty of a demonstration » (B 818). Secondly, there are cases where the grounds from which an item of mathematical knowledge has to be derived are too numerous or too deeply concealed so that an attempt to prove it indirectly may appear to be more promising (*ibid.*).

It is also permissible to employ apagogical proofs in natural sciences. Since all natural knowledge is based upon empirical intuitions generated by means of perception and not by operations of pure logic, it is impossible to substitute in natural sciences the subjective conditions of thought for the conditions of production of intuitive examples or models of thought. The error of subreption can always in principle be guarded against « through repeated comparison of observation » (B 820). Here too, the Kantian requirement on proofs is satisfied.

So it is in the Kantian transcendental proofs of the principles of the understanding. These principles contain either concepts referring to immediate experience or categories. The objective validity and the possibility of the synthesis of the former concepts is ensured by their very formation and,

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4   Cf. *Logik Jaesche*, WW, p. 480; *Logik Politz*, AA, vol. 24.2, p. 527; Refl. 2142, 2178, 2185.
5   Cf. B 817, 820; *Logik Politz*, AA, vol. 24.2, p. 489 : cf. *ibid.*, pp. 561, 584, 749; *Logik Dohna-Wundlacken*, *ibid.*, p. 749; Refl. 2731, 2734).

of the latter, by means of transcendental schemata. The Kantian schemata do in respect of categories the same job which is done by mathematical construction procedures in respect of mathematical concepts. It can therefore be said that, in producing proofs of the principles of the understanding in which categories occur, Kant profits from the special guidance from outside categories themselves. For instance, in transcendentally proving the principle of causality Kant does not pass directly from the concept of that which happens to the concept of a cause. Such a *saltus* would not be in agreement with the fundamental requirement on proofs of synthetic judgments. Kant's special guidance here is, of course, a particular aspect of possible experience, namely, that one which is generated by applying the schema of causality. The transcendental proof of the principle of causality, as well as all other transcendental proofs offered by Kant, proceeds, thus, « by showing that experience itself, and therefore the object of experience, *would be impossible* without a connection of this kind. Accordingly, the proof must also at the same time show the *possibility of arriving synthetically* and *a priori* at some knowledge of things which was contained in the concepts of them » (B 811, my italics).

References

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