

Could Wittgenstein still be held as a father of truth tables?

Poderia Wittgenstein ainda ser considerado como um pai das tabelas de verdade?

“(...) the great question is now: How should a notation [Zeichensystem] be constructed, which will make every tautology recognizable as a tautology in one and the same way? This is the fundamental problem of logic! [Dies ist das Grundproblem der Logik!]“

Wittgenstein, Letter to Russell, 1913

Abstract

A recent discussion on the origins of truth tables as notational means is addressed in this paper. Some authors, engaged in a historical and technical investigation, overlook several philosophical features of Wittgenstein's use of truth tables and its connections with the Tractarian Philosophy of Logic. Here, I discuss the distinction between a use of truth tables as a metaphysical instrument to capture the essence of both language and world and a secularised use, for instance, as a decision tool in propositional logic, without any further substantive philosophical program. The secularized use of truth tables, different from a Tractarian approach, is a use independent of any philosophical account or metaphysical constraint.

Key words: Wittgenstein; Truth tables; *Tractatus*; Tautology; Contradiction; Symbolism.

Resumo

Neste artigo abordaremos uma discussão recente sobre a origem das tabelas de verdade como um tipo de notação. Alguns autores, engajados em uma investigação histórica e técnica, negligenciam várias características filosóficas do uso que Wittgenstein faz de tabelas de verdade e suas conexões com a filosofia tractariana da lógica. Discutiremos aqui a distinção entre o uso de tabelas de verdade como

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um instrumento metafísico para capturar a essência da linguagem e do mundo, e um uso secularizado, como, por exemplo, uma ferramenta de decisão na lógica proposicional, sem qualquer outro programa filosófico substantivo. O uso secularizado de tabelas de verdade, diferente de uma abordagem tractariana, é um uso independente de qualquer abordagem filosófica ou restrição metafísica.

Palavras-chave: Wittgenstein, tabelas de verdade, *Tractatus*, tautologia, contradição, simbolismo

Introduction

At the beginning of the 1990s, Shosky discovered truth-table matrices in Wittgenstein's handwriting in the typed transcript of Russell's 1912 lecture 'Matter: the problem stated'. He then holds them as the first traceable truth tables (Shosky, 1997: 22). Since then, other recent studies have disputed who invented truth tables, both as a logical technique and as a logical device (e.g., Grattan-Guinness 1997, 2005; Anellis 2004, 2012 and Beziau 2012). Illustrative of this discussion, Anellis (2012) defends Peirce as being the first to fully develop a diagrammatical representation for material implication more than 20 years before Wittgenstein's Tractarian truth table.

Nonetheless, this kind of historical approach to the topic of the origin of truth tables very often overlooks at least three crucial points. First, although we may have some proof that other authors have entertained the idea of a tabular representation of truth-functionality, Wittgenstein explicitly held truth tables as *his own* notation¹ and *not* as someone else's creation (see Wittgenstein 1929: 170-1 and Waismann 1979: 80-92). Second, the search for an adequate notational means plays a central metaphysical role in his early philosophy. It is crucial to note that he is, at that time, not merely developing a

1 Ramsey (1923), for example, received the Tractarian truth-table *notation* as a contribution to logic and to philosophy. He clearly takes Wittgenstein's presentation of material implication in the *Tractatus* as original (p. 470). Furthermore, Ramsey held the Tractarian notation as sharing some relevant logical features with a perfect notation: "It may, of course, be doubted whether it is possible to formulate this rule [on logical dependence of propositions] as it seems to presuppose the whole of symbolic logic; but in any perfect notation it might be possible; for example in Mr. Wittgenstein's notation with T's and F's there would be no difficulty." (RAMSEY 1923: 472). See also Ramsey 1927: 164-166.

logical device to prove the validity of some formula. He uses truth tables² in a more ambitious (philosophical) way, namely – aiming at showing the essence of language – than, for instance, Post (1921) did. Third, Wittgenstein's truth tables represent a development of his other tentative symbolisms, as some letters to Russell from 1913 strongly indicate³. In this respect, it is important to note that the *Tractatus* presents at least another equivalent notation (see 6.12034)⁴.

During the Tractarian period, the notational means should not just be a logical tool for mechanically calculating whether a molecular proposition is valid or not. In other words, regardless of which notational means is in question, for the young Wittgenstein, they should all exhibit some *essential* features of language (and then of world). For this reason, we should bring this discussion on the origin of truth tables back to a philosophical context rather than a historical one. As Hacker (1997: 46) states: “what mattered to Wittgenstein, however, was not primarily the decision procedure, but rather what the T/F *notation* shows about the nature of the propositions of logic and truth-functional operators”. The already mentioned recent works on the origin of truth tables systematically neglect that Wittgenstein, at that time, was not merely elaborating some technical device for calculation in logic. Rather, he had a very pervasive philosophical program that should also integrate notational means.

We can take, for instance, the Tractarian passage 5.4711 as an explicit example of what I am calling a *metaphysical* approach to the issue, which makes Wittgenstein's use of it peculiar. If specifying the essence of language means specifying the essence of world and if we are right in maintaining that

2 I refer here to the classical usage of truth tables outside a many-valued background. For further discussion on this topic, see Shramko and Wansing (2011) and Beziau (2012).

3 In fact, by discussing some of these letters, we may understand why Russell held the *Tractatus* (not surprisingly) as a project towards the construction of an ideal language in his historical and controversial introduction to Wittgenstein's first book. This subject was the main issue in several letters between him and Wittgenstein in 1913. In this sense, these letters give further evidence for Shosky's interpretation of his discovery of truth tables drawn by Wittgenstein in Russell's documents from 1912.

4 All decimal numbers in the present text come from the numeration of the *Tractatus* (1918). Although Wittgenstein's first book was published in German in Ostwald's *Annalen der Naturphilosophie* only in 1921, a first complete version was already to be found in the summer of 1918. For this historical development, see *Tractatus'* critical edition, organized by McGuinness and Schulte (1989).

truth-table notation was thought to mirror the alleged essence of language, then by addressing the construction rules of a truth table, we are addressing the logic or essence of world itself: a world constituted uniquely by independent state of affairs (1-1.21). Moreover, in the Tractarian passages 3.342 and 3.3421, Wittgenstein strongly suggests a metaphysical connection between “das Wesen der Notation” and “das Wesen der Welt”.

Accordingly, one may defend that the kind of factual questions of *who* exactly created truth tables and *where, when and how*, which, for instance, are at the core of Anellis’ critique (2004) of Shosky’s findings, are not decisive from a philosophical point of view. Here, it is rather valuable to examine the question of *why* truth tables were developed. We could also ask: to which kind of (philosophical) problem do they represent a reaction? This is the sort of answer we are aiming at. The *Tractarian* truth table is neither a mere mathematical matrix of truth values nor a simple logical technique for capturing truth-functionality; rather, it represents an alternative notational system to grasp the alleged *essence of language and world*. This is the philosophical motivation to assess *why* Wittgenstein’s early philosophy was deeply marked by an engagement in building several notational means, which would work rather as a metaphysical instrument than as a mere diagrammatical device.

In this respect, regardless of Wittgenstein being the truth-table notation’s real father, it is essential to draw attention to this often underestimated fact by the recent literature on the origins of truth tables: to work with truth tables⁵ is indeed to operate with numerous tenets of the Tractarian philosophy of logic. As a result, common attacks against this notational means as being limited, for instance, may also expose direct problems of its image of logic as also being limited. Wittgenstein himself acknowledges, in the last pages of his paper *Some Remarks of Logical Forms* (SRLF, 1929), some patent logical limitation in *his* notation in dealing, for instance, with the logical multiplicity of phenomena⁶.

5 We refer here to the classical usage of truth tables outside a many-valued background. For further discussion on this topic, see Shramko and Wansing (2011) and Beziau (2012).

6 The abandonment of the truth table as an adequate notation may consequently play a crucial exegetical role in understanding, for instance, some problems in Wittgenstein’s intermediary thought. Another problem with truth-table notation is the exponentialization of its rows in function of the number of elementary propositions we are dealing with. This represents a challenge for its inspection.

If one is familiar with truth tables, one is familiar with the peculiar Tractarian metaphysics (and limitations), whether one is conscious of it or not. This form of tabular representation of truth conditions and truth-values of propositions, which is conventionally advanced today in handbooks for propositional calculus, was named in the Tractarian period as *WF Notation*. In 4.442, for example, Wittgenstein presents a truth table for that which we currently call a material implication. This passage can be held as a birth certificate of truth tables⁷. This special notation should provide a means by which, in one movement, one could (a) mirror the alleged deep (and hidden) syntax of language⁸, (b) fully recognise and calculate a radical difference between empirical (*sinnvolle*) and logical propositions (*sinnlose Sätze*) and (c) avoid logical nonsense. These three features, especially (a), compose what I hold as being a metaphysical use of truth tables. By contrast, a secularised use⁹ does not exhibit any of these features, although, using the very same logical devices, a secularised use of a logical tool is arguably independent, for instance, of any account of the essence of world.

7 Of course, there may be other birth certificates if we do accept that they were developed, in different levels, quite independently by different authors such as Boole, Frege, Peirce, Schröder, Łukasiewicz (for an impressive tentative list, see Anellis 2012). A natural development of the present paper would be, for instance, a direct comparison, both with a philosophical and a technical approach, between Post's truth tables (1921) and the *Tractatus*' truth tables developed during the same period but independently. Post's use of the truth table seems to be a perfect example of what we call a secularized usage because it is deprived of metaphysics and of any substantive philosophical program about the essence of language and world. In this paper, I concentrate my exegetical efforts on the metaphysics of truth tables using a Tractarian background. While Wittgenstein may not have been the first philosopher to conceive of a truth table, he was most likely the first to use it in association with a substantive philosophical program on the nature of language and the essence of the world.

8 This is not to be held in a Chomskyan sense but in the early analytic philosophy tradition sense: legitimate propositions have one and only one logical form; this is hidden and very complex. As a result, philosophers should engage in discovering this form by logical analysis. (For that, see, for example, Frege 1918; Russell 1918; or the Tractarian passage 4.002).

9 I do not intend to connect metaphysics with religion here. I am simply using the term "secularized" regarding some logical devices as a analogy to stress that they could be legitimately held as displaying no mandatory relation to any metaphysical account or substantive philosophical program about the essence of world. Ultimately, metaphysics, even such a peculiar one as found in the *Tractatus*, can be held as irrelevant for understanding and applying logical machinery. This autonomy of logical systems, in this "secularized" perspective, is also an insight that Wittgenstein himself developed in the 1930s. There is really nothing sacred about logical practices. However, it is difficult to see how Wittgenstein's account of logic in the *Tractatus* could be independent of its ontology of elementary *Sachverhalte* and an absolute and unique logical space.

For the present investigation, I divide the text into three sections. In the first, I acknowledge a general Leibnizian heritage in what I call the Tractarian metaphysical approach to a symbolism. The second section offers, say, philosophical evidence for Shosky's discovery by exploring Wittgenstein's urge to (radically) separate logical propositions from empirical propositions, via a perfect notation, without using degrees of evidence as a criterion. The final section examines some differences between a metaphysical use of the truth table, as exhibited in Wittgenstein's early philosophy, and a secularised use, focusing on some logical features such as truth-functionality, bipolarity, logical independence of elementary propositions and the central role of tautologies.

1. Reassessing an influential metaphysical view about symbolism

We shall accommodate here the Tractarian project regarding notational means inside a more comprehensive Leibnizian perspective about the philosophical role of symbolic systems. We are not claiming any direct influence or heritage but rather a pervasive affinity in dealing with symbols. This is to show that the young Wittgenstein is not alone in his metaphysical use of a symbolism. Such acknowledgment is not to be found in the recent literature about truth tables. This view holds that our symbols, ruled in an integrated system, can be firm threads to the nature of things rather than inevitable obstacles to them. In 1678, for example, in a letter to Tschirnhaus, Leibniz maintains: "We do not have to fear that the contemplation of symbols will lead us away from things, but, on the contrary, it conducts us to their inner side"¹⁰.

Our symbolism should indeed mirror the essence of things: by exploring certain signs, we can indirectly grasp the nature of a domain. Accordingly, a symbolism is not of supplementary importance in philosophy, but it is essential for philosophical activities. We can argue that the Tractarian project fits well within the context of this sort of (philosophical) language optimism, where the elements of a notational system can and should be adequately articulated to lead us to the interior of language, which could, in turn, express the logical structure of world so that our philosophical confusions dissolve and discussion comes to a halt.

¹⁰ "Nemo autem vereri debet, ne characterum contemplatio nos a rebus abducat, imo contra ad intima rerum ducet." (Leibniz 1918, p. 461)

This preliminary understanding enables us, for example, to make sense of certain lines of Wittgenstein's criticism of Russell and Frege in the *Tractatus*, when he argues that their notation could not rule out all mistakes:

To avoid these [philosophical] errors, we must employ a symbolism which excludes them, by not applying the same sign in different symbols and by not applying signs in the same way which signify in different ways. A symbolism, that is to say, which obeys the rules of logical grammar—of logical syntax. (The logical symbolism [Begriffsschrift] of Frege and Russell is such a language, which, however, does still not exclude all errors.) (3.325).

Here, it is clear that the young Wittgenstein believed that a symbolism should both grasp the rules of logical syntax, held as unique at this time, and avoid logical errors. By writing “language which does not still exclude of all errors”, Wittgenstein most likely meant, on the one side, the possibility, discovered by Russell in approximately 1901, of building inconsistent *Sätze* inside *Grundgesetze's* system, which led Frege to consternation and his system to trivialisation. On the other side, we have the possibility, in *Principia*, of holding some propositions as logical, which in fact should not belong to logic¹¹.

It is also noteworthy that the above Tractarian passage marks the unique use of the word “*Grammatik*” in the entire book. This notion has undoubtedly played a central role in the development of his philosophy (see, for instance, Hintikka and Hintikka, 1986). In 3.325, “*Grammatik*” revealingly appears articulated with “*der logischen Syntax*” (not several, but one) and the search for a *Begriffsschrift*. The choice of this word, in a context where Wittgenstein could have used “*Zeichensprache*” or “*Notation*”, does not seem to be accidental. In this context, this direct mention of a “*Begriffsschrift*” explicitly associates his Tractarian logical inquiry with Frege's and Russell's search for adequate notational means to explore and advance logic. It is crucial to note here that a search for a correct symbolism meant *inter alia* an attempt at systematically avoiding logical nonsense and philosophical confusion.

Accordingly, Ramsey, in his historical 1923 review of the *Tractatus*, recognises the Tractarian truth-table notation as an original improvement in the expression of dependence between propositions and their operators in comparison with the notation of *Principia*. That Ramsey held the Tractarian truth-table notation as an original contribution to logic is consistent with

11 We will come back to this in the next section.

the already presented fact that Wittgenstein held it as being *his own* notation and with Shosky (1997) findings. Moreover, Ramsey, in the very reception of the *Tractatus*, correctly held its truth table as an *alternative notation* with a clear philosophical aim and not as a *mere mathematical device or a logical technique*. This is compatible with the Tractarian approach to truth tables I am defending to make sense how truth tables capture several features of the *Tractatus*. Despite Ramsey's appraisal, Wittgenstein, in his *Nachlass*, expresses some consternation because Ramsey could not really fully understand the crucial (philosophical) stress he put on symbolism:

Ramsey neither understands the value that I set on a particular notation nor the value that I set on a particular word because he does not see that a whole point of view [Anschauungsweise] of an object is expressed through it, the angle from which I now consider a thing. The notation is the ultimate expression of the philosophical intuition [Philosophische Anschauung]. (Wittgenstein 1994: 165, my translation and emphasis).

Although Wittgenstein did not mention Leibniz in this context, his approach to symbolism is relevantly close to the presented Leibnizian ideal of symbolism, most likely because of his direct dialogue and dispute with Logicism¹². The young Wittgenstein rightly followed the logicist tradition represented *inter alia* by Russell's idea that the grammar of everyday language does not necessarily reflect the logical syntax of language and that this can lead us to erroneous analysis¹³. The search for a notation naturally accompanies the Tractarian project of an "excavating" logical approach to language: logical form is usually much more complex than we think and is hidden in the "subsoil" of language. As result of this perspective, we would need a prospective means to excavate this logical form to bring it to the surface. In this context, it is natural to require a notation to be the safe thread and criterion for a more adequate logical analysis¹⁴.

12 For further discussion on this articulation between Leibniz and Logicism, see Peckhaus 1999.

13 This is to be found in the kind of logical analysis proposed in *On Denoting* (1905). Russell's innovative method of dissolving ontological mistakes and philosophical illusions, such as the Meinongian theory of inconsistent objects, through logical tools and analysis deeply influenced several authors in analytic philosophy (the young Wittgenstein as well). For that, see 4.002 and 4.0031. This seems to be the first and only positive mention of Russell's work in the whole *Tractatus*.

14 For further discussion on this point, see Hintikka and Hintikka 1986, chapter V.

It is also important to emphasise the distinction between excluding errors by ameliorating the notational system rather than by improving the language itself. According to the *Tractatus*, language itself should not be corrected; it is logically perfect the way it is (5.663). A perfect notation should, then, simply bring to the surface this hidden logical form. The project of developing a more suitable notation or an instrument for “logical excavation” is pursued by Wittgenstein from *Notebooks* 14-16 through the Tractarian truth tables up until (at least) the early 1930s, when he tried to access alternatives that could capture the logical multiplicity of phenomena, such as the case of the octahedron in relation to the colour system¹⁵.

It is well known that an important feature of the Tractarian metaphysics of symbolism is that language can represent, in principle, the whole reality, or that facts can always be “remitted” to other facts to depict them, but language cannot represent what makes this representation possible, i.e., the logical form shared by represented and representation (4.12 and 4.121). Accordingly, it is legitimate to hold that the Tractarian distinction between *saying* and *showing* comes, among other things, to fulfil the role of clearly determining the distinction between representation through description and representation *via* notations. Wittgenstein argues that what can be shown cannot be said. Because saying corresponds to the sphere of bipolarity, that is, the sphere of sense and contingency, it cannot cover the necessary sphere of language, for example, the sphere that should be *shown* by an adequate notation. For instance, that all propositions of logic are tautologies, that meaningful propositions should be contingent and that language works exclusively truth-functionally should all be *shown* by an adequate notation, although they cannot be *stated* by language. Something necessary within language cannot be formulated meaningfully. Rather, it should be evidenced by its syntax. This is clear and desirable in this Tractarian period: (the unique) syntax of language that captures the logic of world should be systematically exhibited by a notational system.

The Tractarian search for a correct notational means, or a perfect one, as Wittgenstein often maintains in *Some Remarks of Logical Form* (1929, SRLF), for instance) incorporates a Leibnizian view such that it can be used as a leitmotif to understand his image of logic. As a result, it is not an accident that the truth-table notation shows an appeal to logic as an exclusively neutral field that is completely combinatorial (4.27, 4.28 and 4.442), where no

15 See, for example, Wittgenstein 1994, pp. 57-60. For further literature on this topic, see Silva (2016a and 2016b).

possibility of combination is excluded (2.0121¹⁶) and no hierarchies are expected (5.556). The truth table incorporates the whole paradigm of truth-functionality (5 and 5.1) and the neutrality of logic (5.551).

2. Wittgenstein's philosophical introduction of truth tables

By applying this Leibnizian ideal on the metaphysical character of correct symbolism in examining, for instance, *Notebooks* 14-16, we find Wittgenstein's first entry about the notational elements of a language on 26th November 1914¹⁷. This passage shows a commitment to the belief that an essential feature of any legitimate language should be shown in an appropriate notation. In this particular case, it is demanded that an adequate notation should show the internal articulation between positive and negative propositions. A correct logical notation should put truth and falsehood at the same level¹⁸. This relationship between essential aspects of language and the expressive capacity of a notation is also evidenced in annotations of 29th November 1914 with respect to the superfluous nature of identity in a notational system, a typical Tractarian issue (3.323, 5.4733, 5.5301, 5.533).

The demand that the rules of a notation should prevent us from the formulation of pseudo-problems is an old idea in Wittgenstein's philosophical journey. In response to Russell's letter of 30th October 1913, he makes clear that the T-F schemes are articulated "genetically" with another tentative notation developed at this time: "(...) If you had only remembered the WF Scheme of $\sim p$ you would never have asked this question (I think). In fact all rules of the ab symbolism follow directly from the essence of the WF Scheme" (1961: p.123). This is Wittgenstein's (insolent) answer to Russell's question: "If apb is the symbol for p , is bpa the symbol for $\sim p$? And if not, what is?" (Wittgenstein 1961: 125). The talk of "an essence" of a symbolism in the above passage is also noteworthy, as it may firmly orientate one's philosophical questions about logic.

16 In his early philosophy, Wittgenstein often states that logic should deal with *all* possibilities and that *all* possibilities are its facts.

17 Wittgenstein 1961, p. 33

18 This logical tenet is also to be found in Wittgenstein's conversations with Schlick and Waisman from January, 1930: "A negative proposition confers the same multiplicity upon reality as a positive one" (Waismann 1979: 85). Or: "Obviously the case is thus: if ' a ' is possible, then ' $\sim a$ ' is possible too" (id. ib., p.90).

In this historical context, this letter to Russell from 30th October 1913¹⁹ strongly suggests that Wittgenstein was teaching Russell the use of *his* notational means rather than being taught by Russell, as Russell is clearly asking Wittgenstein how to operate and interpret some features of the discussed notation. Wittgenstein was searching for a proper logical means in the pre-*Tractatus* period, and the development of a more adequate notation follows the very development of his ideas.

This letter to Russell represents further historical evidence for Shosky's findings and interpretation on the origin of truth tables as a Wittgensteinian development (Shosky 1997: 22), despite Anellis' criticism that Peirce should be taken as the real father of truth tables (2004, 2012). In these works, Anellis advocates that Peirce should be considered the first to build a diagrammatical device for expressing truth-functionality. Nevertheless, if Wittgenstein is not the father of truth-table notation, we would have a historically meaningful consequence: Ramsey and Russell evidently did not know Peirce's arguable priority because no one mentions Peirce by their public reception of the *Tractatus*, particularly when they are directly discussing the Tractarian symbolism. If Anellis is correct in his historical enterprise, this would mean that some of the pillars of analytic philosophy surprisingly overlooked Peirce's priority.

In another letter to Russell from November 1913, Wittgenstein highlighted the tentative nature of his *ab* notation or symbolism²⁰, clarifying that it was an ongoing search. He was explicitly searching for a "final notation" for capturing logical truths and, accordingly, presented some alternatives to Russell.

I beg you notice that, although I shall make use in what follows of my ab notation, the meaning of this notation is not needed; that is to say, even if this notation should turn out not to be the final correct notation what I am going to say is valid if you only admit - as I believe you must do - that it is a possible notation. (Wittgenstein 1961, p. 127).

19 This letter is documented in *Notebooks* 14-16, Appendix III, pp. 122-124.

20 It is outside the scope of our paper to explore the historical and technical articulation of this pre-Tractarian *ab* notation with the Tractarian truth table. However, if we compare some of Wittgenstein's letters from 1913 (for instance, Wittgenstein, 1961, p. 123) with some passages in the *Tractatus*, we shall see that the *ab* notation mentioned in these letters to Russell is strikingly similar to the "unsere Notation" presented in 6.1203. Both should be an "anschauliche Methode", just like truth-table notation, to show that some complex propositions are tautologies.

The main point seems to be that in all these possible notations, even if they were not the final ones, as Wittgenstein acknowledges, some propositions, which Russell believed were logical truths, such as the axioms of infinity and of reducibility, would be *shown* as not being necessarily true, and, hence, not belonging to logic. In this letter, we identify a clear belief in the possibility of a typology of propositions made *mechanically* by a more adequate notation compared with the *Principia*. In a proper notational means, Wittgenstein could determine without doubt when a proposition belongs to logic or not, being then a contingent (or meaningful) proposition. In this proper notation, we would have, as a result, the ultimate (objective) criterion for distinguishing empirical propositions from logical ones, that is, empirical truths from logical ones, without surprises.

Accordingly, Wittgenstein states: “I can sum up by saying that a logical proposition is one the special cases of which are either tautologous – and then the proposition is true – or self-contradictory (as I shall call it) and then it is false. And the ab notation simply shows directly which of these two it is (if any).” (1961: 125). The search for an adequate notation naturally completes, in this sense, Wittgenstein’s criticism of the logical status of some axioms of the *Principia*. This letter to Russell from 1913 is also notoriously consistent with the Tractarian image of logic, shown especially in the *Tractatus* passages 6.1 to 6.13.

To say that a proposition is a tautology is to say that it belongs to logic. To say that a proposition belongs to logic is to say that it is a tautology. If one can calculate whether a proposition is a tautology, or recognise it when it is a long, complex proposition (6.1262), one can fully calculate whether any proposition belongs to logic. This idea is the announcement of the project of the truth-table notation as another decision procedure to be fulfilled in the *Tractatus*. In his letter to Russell in 1913, Wittgenstein delivers a “user-friendly” guide for the use of his ab notation and the interpretation of its results²¹. This is crucial for our investigation on the philosophical origin of the truth table as a notation: with a correct symbolic rule, one could calculate whether a proposition belongs to logic or not. Wittgenstein goes on, in this letter, teaching Russell how to use this notational rule. He provides Russell a test of the “logicality” of some axioms of the *Principia*²².

21 Wittgenstein 1961: 125.

22 Wittgenstein 1961: 126.

Some axioms of the *Principia* would not be able to pass this test, so they could not be held as purely logical because they would be “accidentally” true and not “essentially” true. At this point, it is important to emphasise that the critique begins with the urge of radically separating logical truths from empirical truths. This criticism gives the philosophical motivation of establishing an adequate notation also as a criterion for meaningfulness, as contingent propositions are the only meaningful ones because if they are true, they are “accidentally” true. That is, they should not belong to logic because logic should be composed solely by necessary truth, as the young Wittgenstein saw it. We could have, then, a more rigorous criterion for logic than mere evidence or generality (6.1231-6.1233).

A correct symbolism could substitute the insistence of a very vague (and in some sense mentalist) criterion for logic: the degree of evidence (*Grad des Einleuchtens*). The ultimate criterion for what logic should be brought into objective notational considerations, as can be read here: “Self-evidence, of which Russell has said so much, can only be discarded in logic by language itself preventing every logical mistake. That logic *a priori* consists in the fact that we cannot think illogically.” (5.4731)²³.

Accordingly, in his review of the *Tractatus*, Ramsey (1923) suggests that Wittgenstein was searching for a perfect language:

However, in a perfect language in which each thing had its own one name, that in the sense of a sentence a certain object occurred, would also be shown visibly by the occurrence in the sentence of the name of that object; and this might be expected to happen with regard to all internal properties of senses; that one sense, for example, is contained in another (i.e., one proposition follows from another) might always appear visibly in the sentences expressing them. (This is nearly achieved in Mr. Wittgenstein’s T notation). Thus in a perfect language all sentences or thoughts would be perfectly clear. (p. 476, our emphasis)

There is no problem in speaking, as Ramsey does in the passage above, of a perfect language in the *Tractatus* if we hold it as a search for a perfect notation to express the already perfect logical syntax of current language (5.5563). Wittgenstein himself strongly indicates in SRLF that he was still searching for

23 This passage is echoed in a revealing criticism to Frege in 6.1271 about advocating evidence as a criterion for logic, which we will discuss in a further section.

a “perfect notation” by the time of his return to philosophy in 1929 because his notation was unable to rule out nonsensical constructions²⁴ (p.170). The search for a better expression of certain logical relations through a more adequate artificial language becomes clear when we assume that if a satisfactory notation were to be developed, it should guide us in the way of contemplating and acknowledging (exhaustively) logical truths (in the *Tractatus*, tautologies as shown in 6.1). Thus, one would not need to postulate logical truths. One should only be able to build a correct system of signs, the construction rules for which could systematically reflect the syntactic rules of our language or of any possible language. Such an approach is clearly advocated by Wittgenstein in the *Tractatus*: “Whence it follows that we can get on without logical propositions, *for we can recognise in an adequate notation the formal properties of the propositions by mere inspection.*” (6.122, my emphasis).

According to this symbolic ideal, one may have every logical truth if one construct a proper notation: “It now becomes clear why we often feel as though “logical truths must be “postulated” by us. We can in fact postulate them in so far as we can postulate an adequate notation” (6.1223). We would then need an ideal notation to reveal the deep syntactic rules of our language so that linguistic confusion could be systematically avoided because a poor application or use of language could be automatically denounced by *inspecting* the use of our notation. This should be clearly *visible* through the misuse of the notational system²⁵.

In the *Tractatus*, the adequacy of the structural symbolic power of a symbolic system in relation to the symbolised domain appears in the sequence of passages 3.33 - 3.334. The notational system does not need to address the meaning or semantics of its signs: it needs only to assume that these must stand for something. Through its descriptive power, the symbolism can

24 Ironically, the same kind of critique he made against Frege’s and Russell’s notations in 1918, presented in the Tractarian passage 3.325, can be used against his own notation. An adequate notation should make it impossible to judge nonsense (cf. 5.5422). His truth-table notation, as he acknowledges in his SRLF (1929, p.171), is unable to do that. For further discussion on this topic, see Silva 2016a and 2016b.

25 To see how visual aspects in this inspection are important to this notational discussion compare 6.1203 and 6.1223; see also Ramsey 1923, pp. 176-7.

capture the combinatorial possibilities of external elements to the notation²⁶. Wittgenstein believed this to be one of Russell's (many) errors: Russell started to talk about the meaning of signs to establish his symbolism. Subsequently, for Wittgenstein, language is not entirely ineffable. The semantic relations of a language may remain ineffable, but its syntactic structure should be systematically revealed by the rules of a correct notational system. Accordingly, by using the seminal distinction between language as a calculus and language as a universal medium introduced by van Heijenoort in 1967, Hintikka & Hintikka maintain that:

All that a believer in language as the universal medium can do in his or her logic is to speak of the words and other symbols of language, abstracted from their semantical function. In brief, he or she is led to a purely formalistic conception of logic by a syntax of language as purely formal enterprise can be motivated in two entirely different ways. (...) The inexpressibility is confined to the semantics of our language and its structure. In contrast, the syntax of language can be expressed and discussed in language. And in fact, the views expressed in the Tractatus amount to a strong incentive to study "the logical syntax of language". (pp. 10-11).

Thus, Wittgenstein was forced to take a purely syntactic or formal approach to language because the semantic sphere of the investigation was banned. This partial possibility of meta-language involved precisely the set-up of a notation representing the structural rules of language. We can tackle the rules of formation, and of syntactic articulations of language, particularly when we address the development of a notational system, as logical features are *shown* by the rules that govern the manipulation of signs. Wittgenstein here is quite consistent with his early philosophical enterprise. Because we cannot *state* anything about the essence of language and the world, the attempt to *show* it by the ruled-construction of complex symbols in a more adequate notational means is a natural consequence.

26 This represents an interesting dislocation of the Tractarian picture theory into discussions about notation, as we can also have a kind of "extrapolation" of results from one domain to another. For example, we can see in a *Bild* what should be the case for the *Bild* being true. Here, from the rules of construction of a notational system, we understand the rules of a particular logical domain that is expressed by this notation. In both cases, we demand the same logical multiplicity (4.011) between that which represents (*Bild* or notation) and that which is represented (a state of affairs or a logical domain).

3. On a Secularised Use and a Metaphysical Use of Truth Tables

Several interesting technical features and applications of truth tables in our current handbooks appear in the *Tractatus* (whether directly or indirectly). Such truth table features include the definition of operators by their truth conditions, algorithmic power at the propositional level, completeness²⁷, possibility to test for consistency or for semantic equivalence of some propositions (i.e., when they have the same truth values in all possible interpretations), as well as tests of the validity and invalidity of some arguments (i.e., where the truth of the premises imposes the truth of the conclusion). These are features of a secularised use of truth tables: *it is not necessary to be engaged in any peculiar metaphysical search for the essence of language or world to legitimately explore this logical machinery.*

In general, the truth table is seen as the only possible positive contribution from the iconic (and misleading²⁸) Tractarian logic to contemporary logic exhausted in the notion of tautology (Beziau 2012). In the Tractarian period, Wittgenstein argues that his notation can be an alternative to the notation of the *Principia* to symbolise molecular propositions and to reveal the logic of our language. As Wittgenstein saw things at that time, the truth table itself should be held as a propositional symbol even before being an algorithm (4.44-4.442). Moreover, it is because the truth table is an adequate propositional symbol that it turns out to be an algorithm in the sense that we could

27 As Floyd (2007) states: “The truth table is an imagined diagram of the constraints that would lay into conceptual place a notion of logical consequence. What we may be left with saying is that Wittgenstein vastly underestimated how complicated the idealization and rigorization of the notion of logical consequence would turn out to be. He had the idea of, or instinct for, the completeness of logic, but without any of the techniques.” (Floyd 2007: 211)

28 For example, Wittgenstein holds in the *Tractatus*: “it is the characteristic mark of logical propositions that one can perceive in the symbol alone that they are true; and this fact contains in itself the whole philosophy of logic. And so also it is one of the most important facts that the truth or falsehood of non-logical propositions cannot be recognized from the propositions alone.” 6.113. Compare also his discourse towards a “method” in a letter to Russell from 1913 (p.125, 1961). In a sense, his image of logic marks both the success and the failure of the *Tractatus*. If logic happens to be fully reduced to tautologies and one develops a decision procedure to recognize them by purely mechanical means, we can understand why Wittgenstein really thought he solved all problems (*im Wesentlichen*) in his first book. It is highly controversial to hold Church’s Theorem as a technical refutation of the Tractarian thesis that every logical propositional could be, by the symbol itself, recognized as such. To what extent can a technical result refute a philosophical system? Perhaps it can indeed refute only the part of a philosophical system that presents technical intentions. Nonetheless, I believe that, if confronted with this alleged refutation, Wittgenstein would say that nothing about his philosophy had been properly understood (as usual).

always calculate whether a complex proposition is a tautology or not. That is, it was by approaching the truth table as a more adequate logical notation than that from the *Principia*, for example, that Wittgenstein gained this far-reaching possibility of exhaustively classifying propositions as tautologies, contradictions and contingencies. This constitutes the ultimate criterion of the expressive legitimacy of complex propositions: if a complex proposition shows itself to be a contingent one, it is a *meaningful* proposition.

A further example of a central feature of the *Tractatus* that is embodied (not accidentally) by truth tables is the strong compositionality or truth-functionality. Every proposition is either elementary or should be analysed uniquely in terms of elementary propositions (*Tractatus* 5 and 6), for if we have a determined base, the complexity shall be fully determined from it. In other words, any complexity should be reduced exhaustively in terms of an elementary base. This can also be seen in the demand for a complete and unambiguous analysis of complex propositions in terms of elementary ones (3.25). This compositionality incorporated in the truth-table notation shows the role of elementary propositions in determining that the analysis has a logical end. We do not need further logical analysis if this limit is reached.

This reveals *inter alia* why we write a proposition as, say, 'p' both in the secularised version of our handbooks and in the Tractarian truth tables (4.24). In so doing, we do not have to be concerned about what is involved inside these propositions. That is, writing a proposition, regardless of what it is about, with a 'p' implies, among other things, that the internal complexity of the proposition (its internal elements and their constitution) is logically irrelevant for the analysis. Speaking in Tractarian terms, if a proposition is composed solely of simple names, it is logically irrelevant to know either what these names are or how many of them there are (5.553 – 5.556). If elementary propositions are indeed legitimate, the complex construed through it should also be legitimate. The combinatorial possibilities of molecular propositions are indifferent to the components that occur within the elementary propositions (5.101 and 6). The construction of complexity would then be entirely combinatorial (4.27, 4.28 and 4.442). Neither in the *Tractatus* nor in our secularised usages of truth tables is any logical relation or interaction among elementary propositions to be expected. The first lines in a truth table represent *all* possible combinations of truth conditions among elementary propositions²⁹.

29 This represents precisely what the Color Exclusion Problem comes to deny. For further discussion on this central topic, see Hintikka and Hintikka, 1986, Von Wright, 1996 and Silva 2016a, 2016b.

We now examine bipolarity³⁰ because it seems to be a guide for notational rules in both the metaphysical and secularised uses of truth tables. The elementary propositions, at the end of analysis, must be capable of being true *and* false. This ‘and’ is crucial for our argument. There cannot be a case in which a proposition has either uniquely true (T) or uniquely false (F) already in the first columns of a truth table as its possible values, where we distribute the truth-values. This “error” would be enough to indicate that either the decomposition or the distribution of truth-values was not completely and/or correctly executed. This “notational fact” mirrors the essential bipolarity of propositions as a criterion of meaningfulness, that is, of propositional sense. In a Tractarian context, T and F are the only possible truth-values ascribable to propositions. They are exclusive and exhaustive – there is no third alternative. Moreover, there is no lack of truth-value, and at an elementary level, each truth-value should appear only once. As a result, from a Tractarian point of view, signs such as the following for an elementary proposition ‘p’ would be logical nonsense and could (and should) be recognised as nonsense just by visually inspecting them: ‘(TTF) p’, ‘(FFT) p’, ‘(T) p’, ‘(F) p’, ‘(⊔) p’ or ‘(TFI) p’, where ‘(⊔)’ would mean a lack of truth value and ‘I’ any third truth value. We use here the “*deutliche Notation*” suggested in 4.442.

Accordingly, the correct sign for an elementary proposition should be ‘(TF) p’. Recall that we are dealing with an ideal in which a notational system should be able to capture the logical rules of language. This reflects the requirement that the alleged deep syntax of language should be systematically reflected by a perspicuous notation. As a desired result, nonsensical constructions should be avoided by a simple inspection of truth-table notation.

In truth-table notation, when we bring a molecular proposition to its atomic basis, we assign the values T *and* F to each proposition, that is, not only true and not only false and no other values. In a fully secularised use, when we set up our truth tables in propositional calculus, we, in several relevant cases, oblige our atomic base to be bipolar, i.e., to exhibit the two

30 In Waismann (1979), Wittgenstein states: “Every proposition is essentially true-false. Thus a proposition has two poles (corresponding to case of its truth and case of its falsity). We call this the sense of a proposition.” p. 94. The radicalization of bipolarity with the abandonment of existent negative facts is the answer to Wittgenstein’s problems with the negation in his *Notebooks* period (cf. Wittgenstein 1961, see entries of November, 24th and 25th 1914). Accordingly, the Tractarian uses of negative facts should be held as being merely terminological (such as in 2.06), as “*sein Grundgedanke*” rejects that negation could add up anything to a proposition. Ultimately, the application of negation should impose no mystery, as its representation in truth tables shows. We will come back to this topic.

possible poles true *and* false. In this sense, it is crucial to note that we do not insist that they be just bivalent: they must be bipolar. They have to show the two poles. To show at least one pole is consistent with being bivalent, but not with being bipolar. Even today, this procedure is still entirely Tractarian. To write down the possibilities of truth-values as T *and* F and not just T *or* F, as the basis of the construction of the truth table, is the very diagrammatic representation of the Tractarian bipolarity. In other words, it represents the requirement for contingency at the elementary basis. That is, the contingency or bipolarity is the ultimate mark (or requirement) of propositional legitimacy at the end of analysis.

It is crucial to note that classical bivalence is weak for the *Tractatus*. Bivalence requires that every proposition must be true or false. In terms of legitimacy of propositions, the bivalence makes no relevant distinction between a tautology and a true proposition or between a contradiction and a false proposition. Both tautologies and contradictions belong to the language syntax (4.46). Now, if we presuppose that propositions must be able to be true *and* false, as the tautology is a proposition that, by definition, can only be true and a contradiction, or a proposition that can only be false, we have, then, trivially that they would not be, strictly speaking, propositions. That is, if in addition to bivalence we require bipolarity, tautologies cannot be propositions because they cannot be false. Conversely, contradictions cannot be propositions either because contradictions can never be true. Bipolarity is a much stricter criterion for propositional sense than bivalence. Bivalence states that any proposition is either true or false while bipolarity maintains that any atomic proposition must be capable of being true and false. Stating something, a bipolar proposition, is to set the subject within this expectation of truth *and* falsity. Nothing resembling a legitimate and necessary proposition (with sense) can exist. In fact, calling a proposition legitimate was redundant for the young Wittgenstein. By definition, propositions should always be legitimate. This is clearly distinct from the Fregean view (5.4733).

More specifically, with regard to the status of logic, we still have the exegetical superiority of holding truth-tables as a notational means. See the claim that propositions of logic are tautologies (6.1). In his review of 1923, Ramsey emphasises two achievements: i) to recognise that logical truths are tautologies and ii) to build up a notational system in which we separate them from empirical truths:

So every proposition is a truth-function of elementary propositions, and many differently constructed propositional signs are the same proposition because, expressing agreement and disagreement with the same truth-possibilities, they have the same sense and are the same truth function of elementary propositions. (...) There are two extreme cases of great importance; if we express disagreement with all the truth-possibilities we get a contradiction, if agreement with them all, a tautology, which says nothing. The propositions of logic are tautologies and to have made clear this, their essential characteristic, is a remarkable achievement. (p.471, our emphasis)

As another consequence of logic being built up through tautologies is that it should be objectively recognised by examining the symbol itself, where a truth table could be held as a decision procedure, as, for instance, Drenben & Floyd (1991) defend. This means that if our image of logic is dominated exhaustively by tautologies, we may consequently have a mechanism of fully ascribing a proposition to logic or to the factual descriptions. The completeness in the exhaustive typology of the propositions (tautological, contradictory, contingent) can be read as a strong Tractarian thesis. In line with the young Wittgenstein, we could hold that the correct classification of syntactically well-formed propositions would be tautological, contradictory and *legitimate ones* because contingency was a criterion for determining the propositional legitimacy, i.e., if a proposition could not be true and false or if it failed to present the two poles of truth values, it could not be considered legitimate. Something in it had failed (6.53). The accent in the *Tractatus* was not on contingency, as we mark it in a secularised use, but, as we have seen, in legitimacy, the main test for which was bipolarity.

Another relevant point that usually receives attention from Wittgenstein scholars, but not from the recent discussion about truth tables, is that for him, logical operators should denote nothing in reality (*sein Grundgedanke* cf. 4.0312). Logical operators should not affect the logical multiplicity of propositions, the bars in the truth table, or the parentheses in conventional notations (4.441, 5.46-5.4611). Logical connectives are, rather, abbreviated signs. Contrary to a typical Fregean realism (6.1271), logical connectives would not replace anything in reality, as they would only express possible combinations that were already informed in the elementary propositions. It is not an accident that all usual logical connectives can be defined by the truth-table notation. Reading this thesis in the *Tractatus*, we can clarify, for instance, the functioning of denial. As Wittgenstein categorically states, for instance, in

5.2341, the negation should invert the sense of a proposition. This should happen because negation simply inverts the truth conditions of the denied proposition. This alleged trivial fact about any proposition is clear in the truth table³¹. It expresses the negation of a proposition '(T,F) p', as '(F,T) p'. This representation of negation *shows diagrammatically* what happens with p when p is denied. Negation should then invert the sense of p by inverting its truth conditions. What made p true before the negation turns out to make it false after it; what made it false turns out to make it true.

Arriving at a final positive topic explored by us in the *Tractatus*, we have a strong (and controversial) assumption of the independence of elementary propositions. An elementary proposition can never exclude another. Exclusion appears only at a level of complexity where there is an operation (or, more precisely, the repetition of an element and its denial). In this way, because there are no operators in the elementary basis of language, it cannot contain exclusions. According to 5.1241, every proposition that contradicts another denies that proposition, i.e., there are no propositions that contradict each other or exclude themselves, without entailing a negation. In this way, the logical product of p and q does not generate a contradiction if p and q are elementary. In other words, if the logical product generates contradictions, p and q are not elementary. The logical exclusion is a clear case of a negative criterion for evaluating whether propositions are elementary or not (6.3751). This is a presupposition about the truth tables that is also made obvious by our secularised practices: elementary propositions must be logically independent of each other. It is not by chance that this “notational fact” fully represents a well-known Tractarian feature. In the clear case of conjunction, we must have as the last column “TFFF (p, q)”. Nothing like “FFFF (p, q)” may be accepted in a conjunction of elementary propositions. This would be sufficient to indicate that at some level (even, for example, if we do not know exactly which one) the propositions are not elementary because they still comprise some form of exclusion. At the elementary level, there should be no exclusion. The propositions must therefore be independent (this idea is stated in 5.134 and its ontological counterpart, 5.135). As above, p and q can only be excluded truth-functionally if q can be analysed in terms of a denial

31 Ramsey (1927), in discussing the meaning of “not” and how p and not-p should be related in a truth-functional way, states: “This could, of course, only be used as *definition* of “not” in a symbolism based directly on the truth-possibilities. Thus in the notation explained on page 95 of Mr. Wittgenstein’s *Tractatus Logico-Philosophicus*, we could define “not-P” as the symbol obtained by interchanging the T’s and blanks in the last column of “P.” (p. 166).

of p or if p can be read in terms of the negation of q . In the *Tractatus*, the exclusion is the hallmark of a disobedience of essential bipolarity of language: a state of affairs must be able to be the case and also not be the case.

Conclusion

The search for notational means is not just something important or adequate when talking about the young Wittgenstein's Philosophy: this is essential for his philosophical program. To operate with truth tables is to operate with several tenets of the Tractarian image of logic, even if Wittgenstein was not the first to think of it as a logical device or as a mathematical matrix of truth values. It is indeed by the privilege of this Tractarian notation in the conceptual framework that we can understand several tenets of his early project as well as explain the strong *metaphysical* commitments of the truth table in its Tractarian origin. This metaphysical commitment, as an account of the essence of world and language, is unfortunately not contemplated by the secularised use of truth tables dominant in our manuals of logic and in the recent discussion on its origins. Wittgenstein did not see truth tables only as a means of defining the logical operators or an algorithmic possibility of an exhaustive typology of complex propositions in a tautological, contradictory and contingent sense. He also saw them as a notational means: as a more adequate expression of propositions. For him, truth tables were an alternative (and superior) notational means to other *Zeichensprachen* or *Begriffsschritte*.

In fact, Wittgenstein thought of truth tables as a *metaphysical tool* that should give us an arguably exhaustive map of the articulations of a logical space thought of as absolute, eternal, without competitors and shared by both language and world. I hold these Tractarian tenets as metaphysical because they are all based on an alleged unique essence of language and world. While Wittgenstein may not have been the first philosopher to conceive of a truth table, he was most likely the first to use it in association with a substantive and far-reaching philosophical program on the nature of language and the essence of the world.

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